Perturbation on Bjorken Flow Background

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But what is a background? And what is Peturbation?

- Background is the generic pattern of fluid flow.
- A perturbation is the deviation from the background flow.

But why are perturbations interesting at all??

Why Perturbations?

It gives theoretical insights.

- Consider the most simple background... Fluid at rest

The fluid four velocity is $u^\mu = (1, 0, 0, 0)$.
($\mu = t, x, y, z$ and $g^{\mu\nu} = diag(1, -1, -1, -1)$)

Why Perturbations?

Consider the fluctuation $\delta u^{\mu}(t, x)$.

- The evolution of this perturbation is determined by equations hydrodynamics.

Let's have a look at the equations of Hydrodynamics...

Basic Equations

The Equations of Hydrodynamics[1, 2]

$$\partial_\mu T^{\mu\nu} = 0 \tag{1}$$

$$\partial_\mu J_i^\mu = 0 \tag{2}$$

where $T^{\mu\nu}$ is the energy-momentum tensor, and J_i^μ is the four current of i^{th} conserved charge.

- We need constitutive relations which relate $T^{\mu\nu}$ and J_i^μ with the energy density, number density and pressure. For ideal fluid

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$
$$J_i^\mu = n_i u^\mu$$

- Also we need an equation of state.
  How do the perturbations we have considered evolve by this equation?

- Let ϵ_0 be the energy density of static fluid, and $\delta\epsilon$ be the energy density corresponding to the fluctuation δu^μ.
- With $\epsilon = \epsilon_0 + \delta\epsilon$, and $u^\mu = (1, \vec{0}) + \delta u^\mu$, and retaining only linear order terms we obtain (for $\alpha = y$)

$$\partial_t \delta u^y - \frac{\eta_0}{\epsilon_0 + p_0} \partial_x^2 \delta u^y = O(\delta^2) \tag{3}$$

- This differential equation is solved by

$$\delta u^y(t, x) = e^{-\omega t + ikx} f_{\omega,k} \tag{4}$$

if ω and k are related as ....

$$\omega = \frac{\eta_0}{\epsilon_0 + p_0} k^2 \tag{5}$$

Now the group velocity of this solution can be calculated

$$v = \frac{d\omega}{dk} = 2\frac{\eta_0}{\epsilon_0 + p_0} k \tag{6}$$

For sufficienty large k, $v > c$ where c is the speed of light.[3]

- This is a problem!!! The velocity of anything cannot exceed the speed of light. What if it does?!!

Acausality Problem

- For all inertial frames

$$\text{Cause} \rightarrow \text{Effect}$$

- But if $v > c$, then it can be shown that

$$\text{Effect} \rightarrow \text{Cause}$$

- But this is unphysical. So the solution obtained of relativistic Navier Stokes equation is unphysical.

- The analysis of perturbations over a uniform background led to the unphysical nature of Relativistic Navier Stokes Equation.
- This motivated to study fluctuations over some different background.
- We chose a particular background called Bjorken Flow.

But why Bjorken Flow? And what is it?

Bjorken Flow

- It is a good model to describe Heavy Ion Collisions.
- According to Bjorken's picture of flow, at a longitudinal distance z away and time t after the collision, the matter should be moving with a velocity $v^z = z/t$.
- Hence the four velocity of the fluid is

$$u^\mu = \gamma(t, 0, 0, z) \tag{7}$$

where $\gamma = t/\sqrt{t^2 - z^2}$

Now let's see the consequences of this statement...

Bjorken's notion of Boost Invariance

- Let us introduce the following two quantities
  - The proper time
    $$\tau = \sqrt{t^2 - z^2} \tag{8}$$
  - Space-time Rapidity
    $$\eta = \frac{1}{2} \ln \left(\frac{t+z}{t-z} \right) \tag{9}$$
- It can be shown that lorentz boost coresponds to a shift in the value of η by constant.
- τ is a lorentz invariant quantity.
- Let us ignore the transverse velocities $v_x = v_y = 0$.
- How do the four velocity components look like in the new coordinates.

Bjorken's notion of Boost Invariance

- The η and τ component of velocity

$$u^\eta = 0$$
$$u^\tau = 1$$

- Moreover energy density, pressure and temperature do not vary with rapidity.
- Let's see how these quantities evolve ...

Bjorken Flow Characteristics

- For the ideal energy-momentum tensor

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} - pg^{\mu\nu} \quad (10)$$

and the equation of state

$$\epsilon = 3p \quad (11)$$

we obtain

- The energy density

$$\epsilon = \epsilon_0 \left(\frac{\tau_0}{\tau}\right)^{4/3} \quad (12)$$

- Entropy varies as $1/\tau$ for ideal $T^{\mu\nu}$.
- Equation of state gives the pressure evolution.
- Thermodynamic relation $\epsilon = Ts - p$ gives temperature evolution.

Differential Equations for perturbations around Bjorken Flow

- We now add a perturbation $u^\tau = 1 + \delta u^\tau$ and see how the energy density fluctuates $\epsilon = \bar{\epsilon} + \delta\epsilon_)$ where $\bar{\epsilon}$ is energy density in Bjorken's flow.
- The three equations for three unknowns $\delta\epsilon$, δu^η and δu^τ.

$$\partial_\tau \delta\epsilon + \frac{4}{3}\frac{\delta\epsilon}{\tau} + \frac{4}{3}\epsilon_0 \left(\frac{\tau_0}{\tau}\right)^{4/3} (\partial_\eta \delta u^\eta + \partial_\tau \delta u^\tau) = 0$$

$$-\frac{\bar{\epsilon}}{\tau}(\delta u^\tau \bar{u_\nu} + \delta u_\nu) + \bar{u_\nu}\partial_\tau \delta\epsilon + 4\bar{\epsilon}\partial_\tau \delta u_\nu - \partial_\nu \delta\epsilon = 0$$

where $\nu = \eta, \tau$

Let's try solving them...

RESULTS

- With or $\nu = \tau$ we obtain

$$\delta u^{\tau} = \delta u^{\tau}(\tau_0, \eta)(\tau/\tau_0)^{2/3} \tag{13}$$

With these we now have two differential equations for $\delta\epsilon(\tau,\eta)$ and $\delta u^{\tau}(\tau,\eta)$

$$-\frac{\bar{\epsilon}}{\tau}\delta u_{\eta} + 4\bar{\epsilon}\partial_{\tau}\delta u_{\eta} - \partial_{\eta}\delta\epsilon = 0 \tag{14}$$

$$\partial_{\tau}\delta\epsilon + \frac{4}{3}\bar{\epsilon}\partial_{\eta}\delta u^{\eta} + \frac{4}{3}\frac{\delta\epsilon}{\tau} + \frac{8}{9}\delta u^{\tau}(\tau_0,\eta)\tau^{-1/3} = 0 \tag{15}$$

- Assuming η-independence of the perturbations we obtain...

$$\delta u^{\eta} = \delta u^{\eta}(\tau_0)(\tau/\tau_0)$$

$$\delta\epsilon = \delta\epsilon(\tau_0)(\tau/\tau_0)^{2/3}$$

Future work

- Get a numerical solution to the equations.
- Calculation in higher order theories.
- Changing the background.

If you want to know more ...

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Thank You