Study of Optical Surface States

7th Semester Lab Report

Experimental and Computational Approach to Optical Tamm Surfaces Plasmons

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Abstract

The reflectivity of a stratified media with a spatially periodic refractive index is analogous to the allowed energy bands of a periodic potential. Similarly, the optical analogy to the Tamm plasmon state in condensed matter physics is the electric field distribution of electromagnetic wave of a particular wavelength which is localized at the interface of a metal and photonic crystal. It is characterized by high absorption and lower reflectivity and transmissivity.

We report the computational principle and methodology to simulate the same. Written in python, the program is designed to compute the electric spectrum give a Photonic Crystal(PhC). The program is well documented and is named "DBR Spectra". We present the computational results spectrum of a given DBR and the field intensity distribution for different wavelengths. The Tamm state and its intensity distribution were determined by simulating a metal layer on the DBR. The Tamm resonance wavelength decreases with increase in the thickness of the metal coating. The same qualitative trend is observed in the case of increasing the angle of incidence.

To demonstrate the usefulness of the program DBR spectra, the spectrum of a photonic crystal was experimentally obtained and was fitted with the simulated one so as to characterize the PhC. Tamm state was obtained for a metal coated DBR and the variation of its wavelength with the angle of incidence was observed.

The future experimental and computational plan for further exploration has been briefly outlined.

Contents

1 Introduction

Analogies between seemingly different physical phenomena roots itself to the similar mathematical structure which describes their physics. Symmetry in nature plays a key role which manifests itself in myriad forms, and give rise to phenomena, completely different and disconnected to an observer, which makes it almost impossible to study each and every one of them separately in their entirety. Therefore we look for symmetry and connection between theories, and natural phenomena, which allows us to understand a wider range of phenomena with fewer theoretical and experimental tools.

Amongst many of these, we discuss here the analogy between a periodic potential in condensed matter physics and its optical analog in terms of refractive indices. This offers a great deal of experimental opportunity. We can take optical systems to do experiment with and determine the results of the corresponding condensed matter experiment.

1.1 Photonic Crystals

Crystals are a periodic array of atoms. The constituting atoms have a definite spatial arrangement, which repeats itself over the entire structure. The atoms in the crystal provides a potential($V(\mathbf{x})$) to the electrons or any particle inside the crystal. The periodic arrangement of the atoms renders the potential to be periodic

$$V(\mathbf{x}+\mathbf{a}) = \mathbf{V}(\mathbf{x})$$

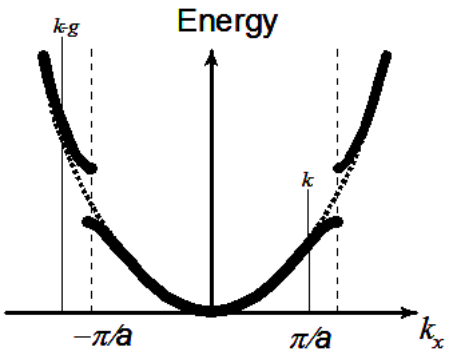
where $\mathbf{a}$ is a constant vector for a given crystal. This translational symmetry, according to Noether's theorem, results in a conserved quantity called crystal momentum ($\mathbf{k}$). The states in the periodic potential, due to Bloch-Floquet is of the form

$$\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\exp(\mathbf{ik}\cdot\mathbf{r})$$

where $u_{\mathbf{k}}(\mathbf{r}+\mathbf{a}) = u_{\mathbf{k}}(\mathbf{r})$.

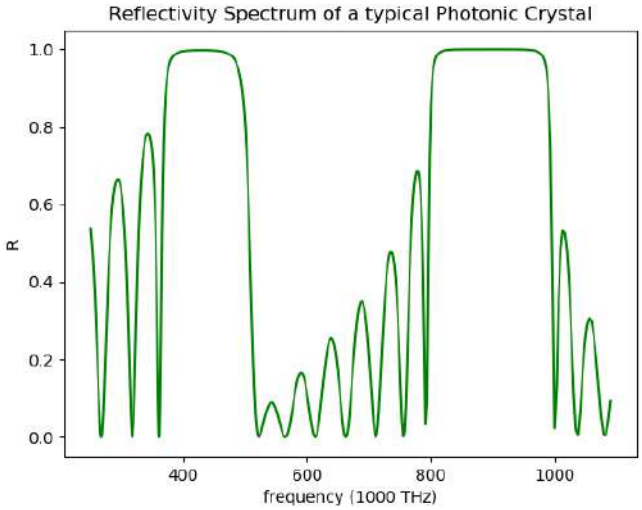
Consider a one-dimensional periodic potential. Owing to the periodicity in real space, there is a periodicity in the crystal momentum space. A particular period in the momentum space is called the Brillouin zone. The Schrödinger equation gives the spectrum for the potential. The plot of allowed energy against

the crystal momentum gives its band structure.



The relation between energy $E(= \hbar\omega)$ and $\mathbf{k}$ is known as dispersion relation. The band structure reveals that some energy states are not allowed. The minimum gap between two energy bands is called a band gap. For further details on the formation of energy bands, we refer to any standard introductory textbook on solid state physics[1][2]. We shall explore the optical analogy of this consed matter phenomena in details.

Analogous to one-dimensional periodic potential, we can have periodic strata of certain refractive indices. These are called Photonic crystal as an analogy with the crystals characterized by periodic arrangement of atoms. Also, they are known as Distributed Bragg Reflectors (DBR). We shall use the term PhC and DBR interchangeably in this report. When white light is incident on it, certain ranges of frequencies are reflected almost completely with reflectivity approaching unity. Following is a plot of typical reflectivity spectrum of a bilayered(a unit cell consisting of two different material) photonic crystal.



Maxwell's equations are also linear, like the Schrödinger equation. The system we shall be working on will have no free charge or current. For the sourceless Ampere's and Faraday's law, we obtain an eigenvalue equation similar to the Bloch Schrödinger equation. Analogous to the Bloch states in quantum mechanics, the Bloch electromagnetic fields can be found[3]. If there is a periodicity in the refractive index

$$n(x + a) = n(x)$$

the Bloch -Floquet theorem applies and only a certain range of frequencies are allowed in the reflectivity spectrum akin to the band structure in condensed matter situation. The frequency ranges over which the reflectivity approaches unity corresponds to the bandgap of the dispersion relation obtained in this case.

Thus the translational symmetry of the refractive index in this situation is the reason of high reflectivity for certain frequency ranges.

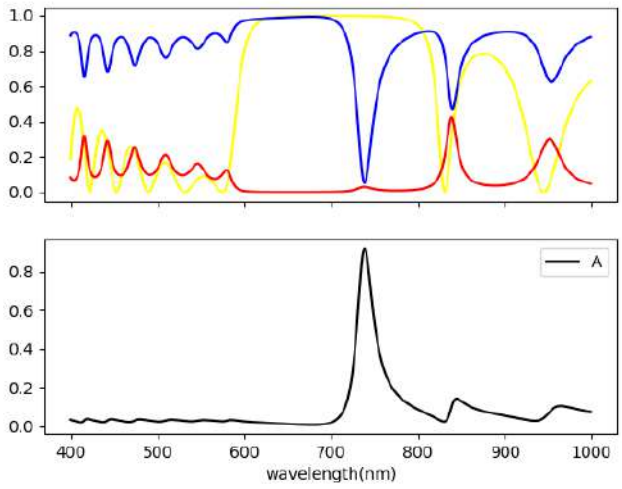
1.2 Surface States

In the previous section we saw that as a consequence of the symmetry we obtain the Bloch-states and bandgaps in the reflectivity spectrum. The Bloch solution for periodic media gets modified once the symmetry is broken[4]. In particular, if the symmetry is broken, by coating a layer of metal onto the surface of PhC,

then a localized electric field of a particular wavelength in the photonic band gap is formed at the metal-PhC interface.

Along the normal to the surface of crystal these solutions decay exponentially, hence they are localized at the surface and are called surface modes. Surface modes which can propagate along the surface area called surface waves while the ones which cannot are called surface states. These states are known to exist since the 1930s. Two kinds of solutions occur as the periodic potential abruptly terminates. The first kind of solutions is the exponentially decaying continuation of the Bloch waves. These are solutions when the periodic media is either metallic or a semiconductor. While the second kind of solution has a localized maximum at the interface which decays exponentially on either side of it. They are the solutions when periodic dielectric layers act as a medium.

In case of metal coated periodic stratified media, it is the localization of electromagnetic field at the metal-PhC interface which decays exponentially on either its side. It is characterized by a low reflectivity as well as low transmissivity at the Tamm wavelength. hence this state is non-propagating and is localized at the interface.



Tamm states in a metal-PhC can be identified by studying it's reflectivity and transmittivity spectra. Consider the spectra given above. The yellow plot is that of reflectivity when no metal coating is there. As the metal is coated, the reflectivity (shown in blue color) increases as expected, but at 739 nm, in the

photonic band gap, we see a dip in reflectivity. The note that at this wavelength the transmissivity is almost vanishing. The absorption plots peaks at this particular wavelength confirming the low values of reflectivity and transmissivity. This implies that this particular state has a localized field distribution inside the metal-PhC system. thus the state at 739 nm is identified as a Tamm state. Field intensity distribution at this wavelength is localized at the interface of the metal and PhC.

1.3 Conventions and Setup

Maxwell's equations are differential equations and boundary conditions are required to determine the exact fields. For electromagnetic waves traveling from one medium to another separated by an infinite plane Maxwell's equations along with the boundary conditions reduce to Fresnel equations. In simple words, polarisation means the orientation of the electromagnetic field with respect to the infinite planar interface. We shall briefly discuss the two polarisations for a single infinite interface, and how the reflectivity and transmissivity differ for the two.

Consider an infinite planar interface passing through the origin and normal to $\hat{\mathbf{x}}$. We assume uniformity in the yz plane. The material on negative x-axis is of refractive index n_1, while that in the positive x-axis is n_2. The wavevector ($\mathbf{k}$) of light is in the xy-plane, with the x-component of $\mathbf{k}$ being positive.

So the angle of incidence is θ_i where

$$\cos\theta_i = -\hat{\mathbf{k}} \cdot \hat{\mathbf{x}}$$

The wavevector depends on the refractive index of the medium in which it is propagating. The magnitude of wavevector of light with angular frequency ω when it propagates through a medium of refractive index n is $k = n\omega/c$, where c is the speed of light in vacuum.

Let light be incident on the interface with angle of incidence θ_i. The component of wavevector perpendicular to the interface is $k\cos\theta_i$, and that parallel to it is $k\sin\theta_i = (n_1\omega/c)\sin\theta_i$. Snell's law ensures the parallel component being constant as light passes from the first medium to the second one

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

The electric field(**E**) and magnetic field(**H**) are perpendicular to the wavevector **k**. If the electromagnetic field is so oriented that the electric field is parallel to the planar interface then it is known as **s** polarisation. In this case, the magnetic field lies in the plane of incidence. On the other hand, if the magnetic field is parallel to the planar interface then it is known as **p** polarisation. In this case, the electric field is in the plane of incidence.

The **E**-field component being parallel to the interface, in case of s-polarisation, is continuous across the sourceless media. For the case of p-polarisation, the same is true for **H**-field.

The coefficient of reflection is the ratio of reflected electric field amplitude to the incident field amplitude. Similarly, coefficient of transmission is defined as the ratio between the transmitted field amplitude to that of incident one. For light going from medium 1 to medium 2 we have [5]

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$t_s = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$t_p = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

These expressions for a single interface is used to compute the same for multiplanar stacks using a method called transfer matrix method. Our computational technique to compute the spectra of a distributed Bragg reflector (DBR) is based on this method.

2 DBR Spectra: The Python program for Simulation

A python program has been written to simulate the required reflectivity and transmissivity of DBR. These are the two experimental quantities which can be measured. Once we simulate the experimentally obtained spectrum, we can obtain the coefficient of reflection and transmission from the program. And hence

the relative phase difference can be obtained for transmission and reflection.

In this section, we shall describe the computational technique used. Also, we shall present a documentation of our program which can be referred to as a manual to use DBR_Spectra.

2.1 Computational Principle and Technique

We report in this section the computational technique used to make the program. We also report a specific computational problem related to the handling of complex refractive when metal is coated, specific to Python(version 2.7) programming language.

2.1.1 Computing Spectra

Having been acquainted with the reflection and transmission coefficient for light going from medium 1 to medium 2, let us extend to the situation where light comes from both surfaces. let the amplitude at the interface be E_{f1}, E_{f2}, E_{b1} and E_{b2}. The latin subscript, f or b, denotes if the amplitude is that of forward or backward travelling wave and the numerical subscript denotes the medium in which light is travelling. These fields are related by the coefficients of reflection and transmission.

$$E_{b1} = E_{f1}r_{12} + E_{b2}f_{21} \tag{1}$$

$$E_{f2} = E_{f1}t_{12} + E_{b2}r_{21} \tag{2}$$

where t_{12}, r_{12} are the respective coefficients for going from medium 1 to medium 2.

If there are N interfaces, labeled from 0 to $N-1$, then the forward and backward going wave amplitude between two adjacent layers are related by a two dimensional matrix. Let v_n and w_n are the forward and backward going wave amplitudes for the n^{th} interface. They are related to that of $(n-1)^{th}$ layer as

$$\begin{bmatrix} v_n \\ w_m \end{bmatrix} = M_n \begin{bmatrix} v_{n+1} \\ w_{n+1} \end{bmatrix} \tag{3}$$

for $n = 1, 2, \ldots N-2$, where

$$M_n = \begin{bmatrix} e^{-i\delta_n} & 0 \\ 0 & e^{i\delta_n} \end{bmatrix} \begin{bmatrix} 1 & r_{n,n+1} \\ r_{n,n+1} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ t_{n,n+1} \end{bmatrix} \tag{4}$$

We want to determine the relative amplittudes of reflected and transmitted waves from a DBR. Taking the incident amplitude to be unity, the reflected amplitude will be r and the trannsmitted amplitude will be t. In terms of matrices

$$\begin{bmatrix} 1 \\ r \end{bmatrix} = M \begin{bmatrix} t \\ 0 \end{bmatrix} \tag{5}$$

where M is

$$M = \frac{1}{t_{0,1}} \begin{bmatrix} 1 & r_{0,1} \\ r_{0,1} & 1 \end{bmatrix} M_1 M_2 \dots M_{N-1} \tag{6}$$

Combining this we get the expression for r and t in terms of the matix elements

$$t = \frac{1}{M_{00}} \tag{7}$$

$$r = \frac{M_{10}}{M_{00}} \tag{8}$$

For a given wavelength we have a particular magnitude of wavevector. This and the angle of incidence fixes the coefficient of reflection and transmission for a single interface, and from there follows the entire matrix method formalism for DBR. So given a DBR, the wavelength of incident light and the angle of incidence, we get unique values of coefficient of reflection and transmission. Taking the square of the magnitude of r and t gives reflectivity (R) and transmissivity (T).The calculations used here are based on the paper titled "Multilayer Optical Calculations by Steven J. Byrnes "[6].

2.1.2 Handling Unphysical Cases for complex Refractive Indices

When it comes to complex refractive indices, the angle of incidence of the corresponding media also becomes complex. Let n_i be the refractive index of the medium and θ_i be the corresponding angle of incidence. Then for each layer Snell's law requires

$$n_0 \sin\theta_0 = n_i \sin\theta_i$$

Thus the value of $n \sin\theta$ will be real for all layers. Hence one would naively expect

$$\theta = \sin^{-1}\left(\frac{n_i}{n_0} \sin\theta_i\right)$$

But the cosine of the angle is troublesome as

$$\cos\theta_i = \pm\sqrt{1-\left(\frac{n_i}{n_0}\sin\theta_i\right)^2}$$

The choice of sign of above depends on our choice of θ or $\pi-\theta$, as both satisfy above equations up to the considered sign of cosine. If not chosen properly, it may lead to unphysical conditions like reflectivity being greater than unity. The above-mentioned paper explains how to resolve the problem in python. it discusses and gives a clear prescription of how to avoid the pathology.

2.1.3 Computing Electric Field Intensity

The position space is discretized into steps of 1nm, and the refractive index has been assigned to each x-coordinate. Then for a given wavelength of light in vacuum, the wavevector for each of the coordinates can be computed. With the angle of incidence known to us, we can further compute the parallel and perpendicular component of wavevector with respect to the infinite planar interfaces. These are the ingredients to compute the propagation matrices from one space point to the next one. If there is no interface then a matrix is assigned to the space point which adds a phase to the field amplitude depending on which medium it is in. If there is no interface then the matrix assigned is

$$P = \begin{bmatrix} e^{-i\delta} & 0 \\ 0 & e^{i\delta} \end{bmatrix} \tag{9}$$

The interface points are those x-points which have a different refractive index at the point just next to it. Apart from assigning a phase propagation matrix the matrix for boundary conditions has to be multiplied with the corresponding P matrix. At the interface

$$\begin{bmatrix} v_x \\ w_x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ n_x\cos\theta_x & -n_x\cos\theta_x \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ n_{x+dx}\cos\theta_{x+dx} & -n_{x+dx}\cos\theta_{x+dx} \end{bmatrix} \begin{bmatrix} v_{x+dx} \\ w_{x+dx} \end{bmatrix}$$

So we assign the following matrix to the interface points

$$D = \begin{bmatrix} 1 & 1 \\ n_x\cos\theta_x & -n_x\cos\theta_x \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ n_{x+dx}\cos\theta_{x+dx} & -n_{x+dx}\cos\theta_{x+dx} \end{bmatrix} \tag{10}$$

Thus we can compute the forward and backward going field amplitudes for all the space points and add both to get the electric field. We compute the intensity and normalize it with respect to the that at initial x-coordinate. This computational technique was based on the MIT lecture notes for layered materials and Photonic Band Diagrams[7].

2.1.4 Metal Coating

We have used Lorenz-Drude model for refractive index of metal. For gold the Lorenz-Drude parameter are

- Collision Wavelength = 8934.20 nm
- Plasma Wavelength = 168.26 nm

using these the corresponding angular frequencies can be calculated. If the angular frequency of incident light is ω, and that of collision and plasma frequencies are ω_p and ω_c respectively, then the square of refractive index is

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2 - i\omega\omega_c}$$

2.2 DBR Spectra:Documentation

DBR spectra is a python program for simulating the spectrum of a periodic stratified media. We document how to use the program in order to simulate a photonic crystal and obtain its spectrum. The metal coating of a particular thickness on the incidence surface or away from it can be simulated. Moreover, angle of incidence can be varied in the simulation.

Though the program is capable of computing spectrum of an arbitrary stratified media, we focus particularly on the ones with bilayered periodicity. The controllable variables in that case are the number of bilayers N, the refractive indices of material constituting the bilayer n_1 and n_2, and their thicknesses d_1 and d_2.

2.2.1 Organisation of Codes

The entire program written in three library files, and one which acts as an interface to do specific tasks like computing spectrum, electric field, varying experimental parameters etc.

We briefly discuss the function of three library files.

1. **core_lib**
2. **E_lib**
3. **E_plot_lib**

The file **core_lib** contains definitions of reflectivity and transmittivity at an interface of two media and angle of incidence. It has scripts to define periodic bilayered PhC and coating the metal. It calculates the transfer matrix for a given set of input parameters.

This script can compute the coefficients of reflection and transmission and hence their phase.

The file **E_lib** contains script to evaluate the electric field. It is dependent on **core_lib** to get the input of photonic crystal and metal coating. The input of the wavelength and angle of incidence comes from the user interface script. As of now only for the 's' polarisation of light, the electric field can be computed. It also has a method to compute the spectra, but the runtime is too long in this case.

The code for plotting of spectrum is in the file **E_plot_lib**. The range of wavelength and other input parameters comes from the user interface program. To compute spectrum it is linked to the core_lib file. The plot of the spectrum is bounded with 'mouse click event'. Once clicked on the plot of the spectrum, the wavelength is taken as input and field computation function in E_lib file is triggered. Then the intensity is computed and normalized to unity. It is this normalized intensity that is plotted by the E_plot_lib scripts.

In order to manipulate the experimental variables and to simulate the spectra, only a few keywords and functions have to be known. In this section, we make a manual of how to use the program and describe all the conventions used and the functions that are enough for simulation purposes.

2.2.2 Defining Input parameters

The script has to be written in a **.py** file and the library functions have to be imported. Also **numpy** and **matplotlib** package of python have to be imported. The code for importing the library files is as follows

```
        import numpy as np
        import matplotlib as plt
        import core_lib as cl
        import E_lib as el
        import E_plot_lib as ef
```

The following code assigns the input parameters to the variables.

```
        n_med =1.0
        theta_i=0.0 #Normal Incidence
        wl=np.arange(550,1210, 1)
        wl_p=168
        wl_c=8934.20
        d_metal=30
        metal_pos='l'
```

n_med is the refractive index of the medium in which the experiment is conducted. It is assigned the value 1.0 since we are doing the experiment with air as the medium (with refractive index = 1). The angle of incidence is assigned to theta_i. The range of wavelength for which the spectrum is plotted is stored in the variable wl. In this code, the wavelength varies from 550 nm to 1210 nm in steps of 1 nm.

The Drude model parameters for metal are plasma wavelength (wl_p) and collision wavelength (wl_c). The values given in the code are those for Gold (Au). The thickness of metal to be coated is given by d_metal.

The variable metal_pos stands for the metal position with respect to light coming from left propagating towards the right. It takes two variables, either 'l' or 'r'. If metal_pos = 'l', then the light is incident on the metal surface. If metal_pos = 'r', then the metal coated to the opposite of surface on which light is incident.

The following code characterises the PhC

```
        #N must be an integer
        N=9

        #n1 and n2 are the refractive indices
        n1=1.5
        n2=2.5

        #d1 and d2 are the corresponding thickness (nm)
        d1=50
        d2=135
```

The following variable is a list which tells the program whether or not to coat the metal. It is an array of five variables

```
        metal_coat=[True, wl_p, wl_c, d_metal, metal_pos]
```

First of all the Photonic crystal needs to be defined. It is done by a core_lib function **n_and_d** It takes input as n1, n2, d1, d2, and N, and returns the list of refractive indices and corresponding thicknesses from left to right.

```
        n_input, d_list=cl.n_and_d(n1,n2, d1, d2, N)
```

2.2.3 Obtaining the Spectrum and Field Intensity

Having defined the input parameters, now the spectrum can be computed using the function **plot_RT_vs_wavelength**. The input it takes is as wl, theta_i, n_med, n_input, d_list, and metal_coat. It returns the list of wavelengths, the phase of coefficient of reflectivity, and transmittivity.

```
        wvl, ph_r, R, T=cl.plot_RT_vs_wavelength(wl, theta_i,
        n_med, n_input, d_list, metal_coat)
```

Since we have the wavelengths and three other physical quantities, viz, the phase of coefficient or reflectivity, reflectivity and transmissivity, we can plot them easily. For example, the following code plots reflectivity spectrum

```
        plt.plot(wvl, R)
        plt.xlabel('wavelength (nm)')
        plt.ylabel('R')
        plt.title('Reflectivity Spectrum')
        plt.show()
```

To get the plot of electric field intensity ('s' polarisation), the code is following

```
        wvl=631  #nm
        ef.plot_field(wvl, theta_i, n_med, n_input, d_list, metal_coat)
```

The functions presented in this document are enough to simulate and study different experimental cases. Besides simulations, we can use this program to characterize a PhC by fitting its spectrum with the simulated one.

The code for spectrum and electric field in 'p' polarisation is under the process of development. It will be updated in further versions along with appropriate changes required by the experiment.

3 Computational Results

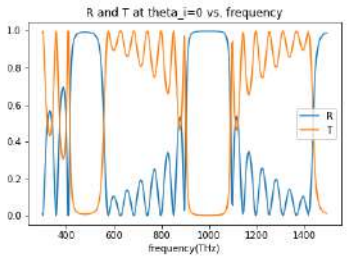
The spectrum gives reflectivity distribution over different frequencies of light. It is depicted plot with reflectivity on y-axis taking values from 0 to 1 versus frequency or wavelength of the light. We obtained some reflectivity spectrum and matched it with plots from available literature[8] to confirm if our program is working properly.

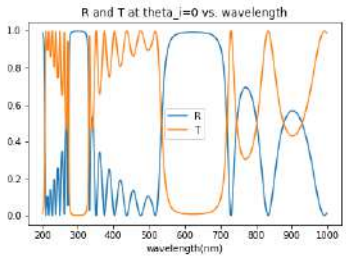
3.1 Spectrum of DBR

The following figures are those for an 8-layered DBR with alternating layers of refractive indices 1.5 and 2.5 of widths 50 nm and 90 nm respectively.

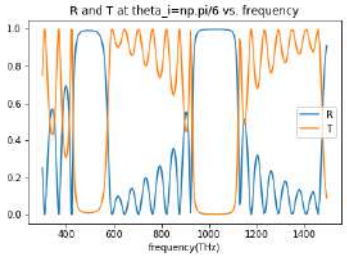
These plots depict the reflectivity and transmissivity as a function of wavelength and frequency for the different angle of incidence.

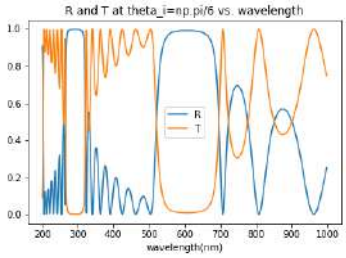
Normal Incidence



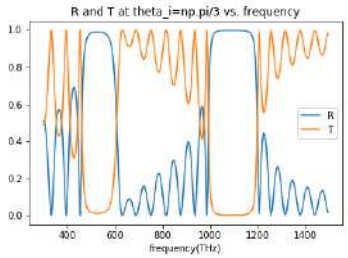


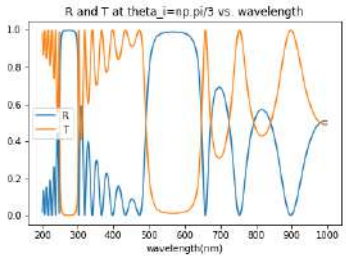
Angle of Incidence $\theta_i = \pi/6$





Angle of Incidence $\theta_i = \pi/3$



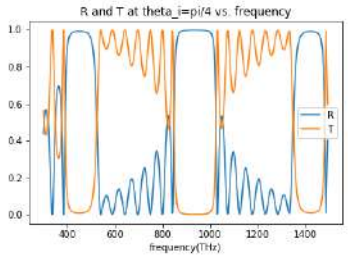


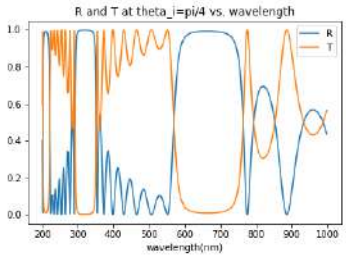
The plots are almost the same for the different angle of incidences. There is a certain region in frequency where reflectivity is very high and hence very low transmissivity. This region an analogous to the band gap (the forbidden region) is the dispersion relation of the periodic potential in solid state physics.

Another DBR:

The following plots are for a different DBR. The layers are of refractive indices 1.5 and 2.5 but the widths are 60 nm and 100 nm respectively.

Angle of Incidence $\theta_i = \pi/4$





These plots matches with those given literature and hence we confirmed our program is working fine.

3.2 Plot of 's' polarised Electric Field

The program is so designed that the light falls on the PhC from the left side. The arrangement of the bilayers in the simulation is following.

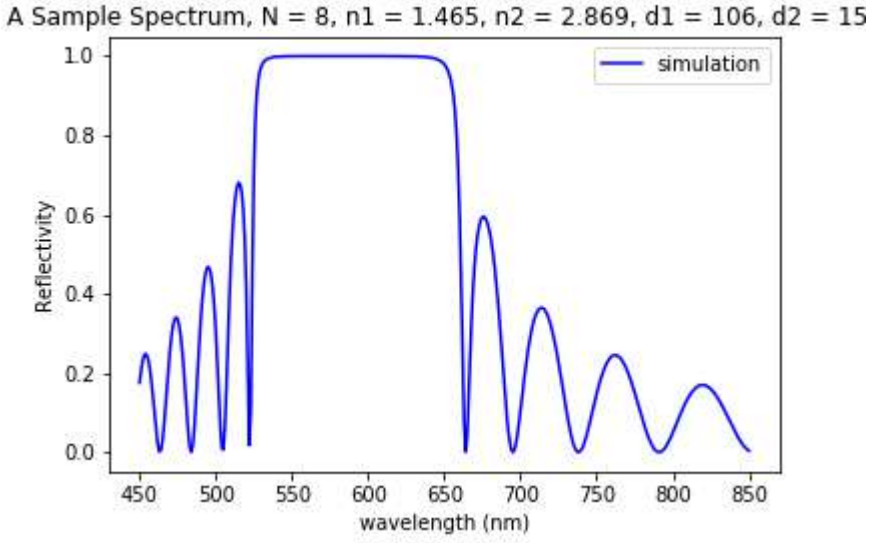
White light$\rightarrow$ $\vert$ n_1 $\vert$ n_2 $\vert$ n_1 $\vert$ n_2 $\vert$ $\ldots$ $\vert$ n_1 $\vert$ n_2 $\vert$

Having obtained the reflectivity spectrum, it is bounded with 'mouse click event' to calculate the electric field calculation at the wavelength on which the mouse is clicked.

3.2.1 Electric field for some Wavelengths

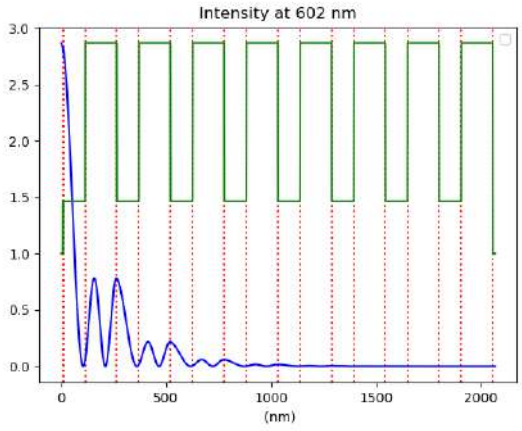
The plot of electric field gives some useful insights as we shall demonstrate below.

Consider the following spectrum.



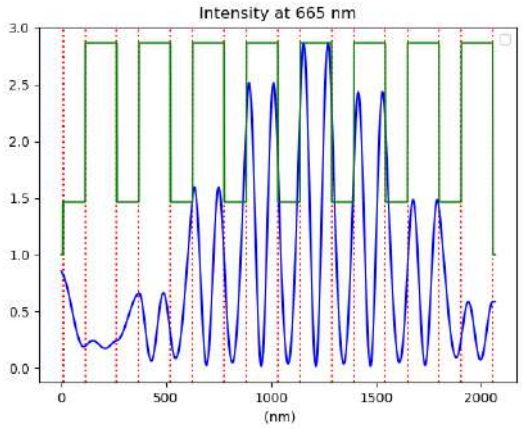
Let us see the Electric field intensity at different wavelengths. For a certain range of wavelengths, the reflectivity is almost unity which is a mark of the forbidden region or the band gap. In the spectrum, dips in the reflectivity are found for certain wavelengths. This means light of these frequencies are not reflected back. While at other wavelengths the reflectivity is neither vanishing nor unity, which implies partial reflection and partial transmission of those wavelengths.

Let us first see the electric field in the band gap, where reflectivity is almost unity.

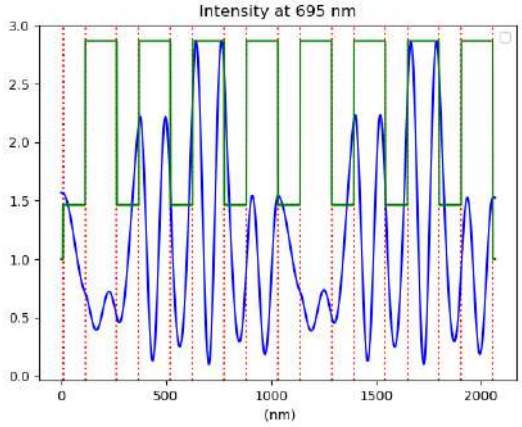


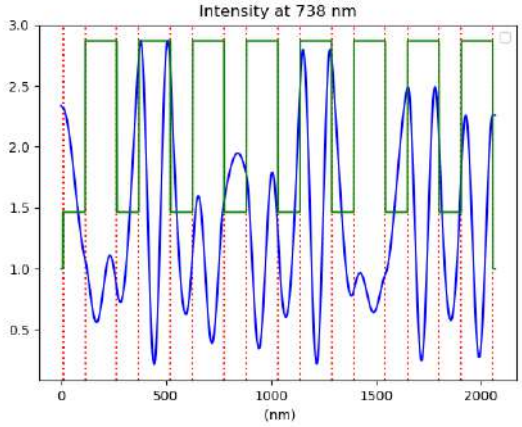
The above plot is the field intensity at 602 nm which is in the band gap of the spectrum. The field is evanescent, which shows that light is not getting transmitted through, and it is getting reflected.

Now let us analyze the intensity of the low reflectivity region. The first minimum after the bandgap occurs at 665 nm. The intensity distribution for this wavelength is shown in the following figure



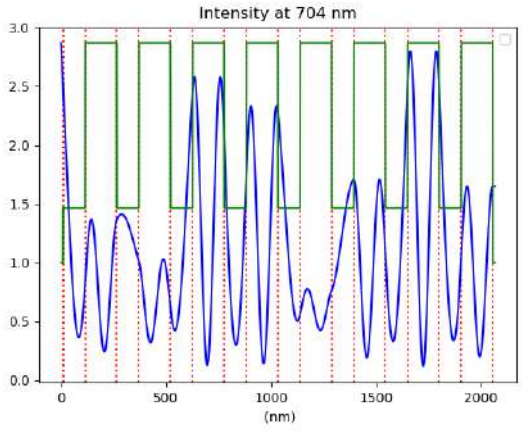
The wavepacket looks like the first harmonic of the standing mode of a vibrating string. Now if we consider the field at second (695 nm), third (738 nm) minima and higher wavelengths at which minima occurs after the bandgap we get the corresponding number of lobes of wavepackets.





At these wavelengths, a normal mode is created in the photonic crystal which accounts for almost perfect transmission of those wavelengths.

The field intensity at any other point looks like following



Note that the field intensity at the boundary of the photonic crystal is non-vanishing. This is the characteristic of those wavelengths whose are not the normal modes of the PhC.

3.3 Tamm State and its Intensity Distribution

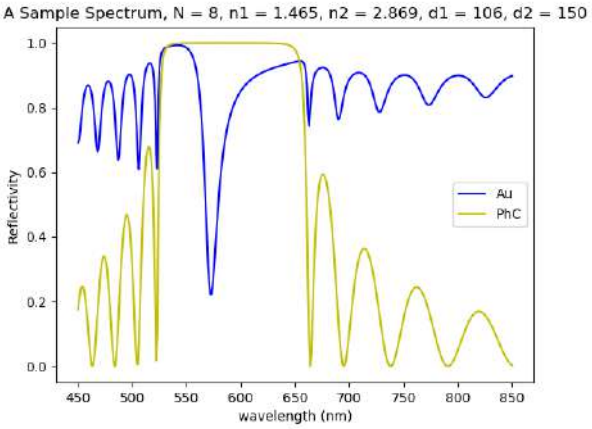
Interesting phenomena occurs if a metal is coated to the PhC. Absorption of a particular wavelength (Tamm state) in the band gap is observed in such cases. But it is not always the case. We discuss the criteria, as obtained from the simulations, for which a Tamm state is formed.

3.3.1 Light incident on the metal coating

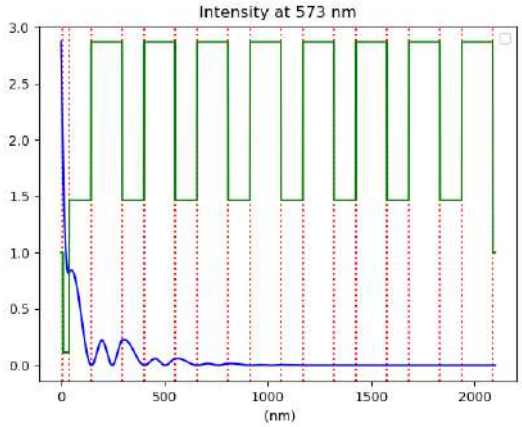
First let us consider the situation in which the PhC is coated with metal on the surface on which light incident. The configuration is as follows

White light$\rightarrow$ | metal | n_1 | n_2 | n_1 | n_2 | ... | n_1 | n_2 |

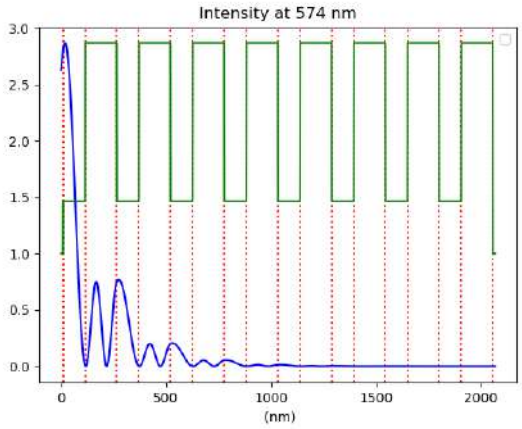
Gold of 30nm thickness is coated on the incidence surface and spectrum was simulated. Following is the corresponding plot of reflectivity. The spectrum for PhC is also given for the sake of comparison.



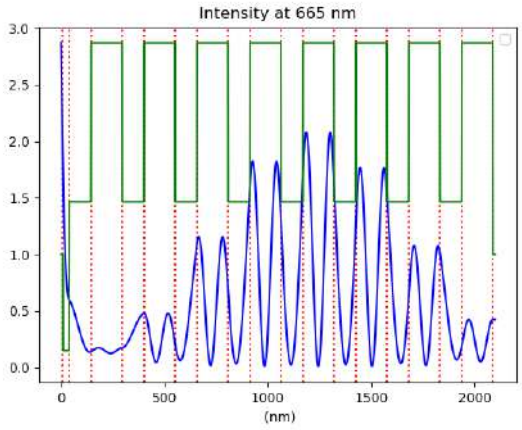
We observe that the metal coating increases the reflectivity since the spectrum is shifted up towards unity. But we observe a dip in reflectivity in the wavelength range which was the bandgap when metal was not coated. Let us look at the electric field intensity at that wavelength (573 nm).



The peculiarity of this particular state is that the reflectivity is quite low and the field is evanescent. That means the photons at this frequency is neither reflected, nor it is passing through the PhC. This state is known as Tamm state, and it is sustained at the interface of metal and PhC. The Tamm state is obtained at 573 nm. If the metal were not coated then the field distribution would look as follows

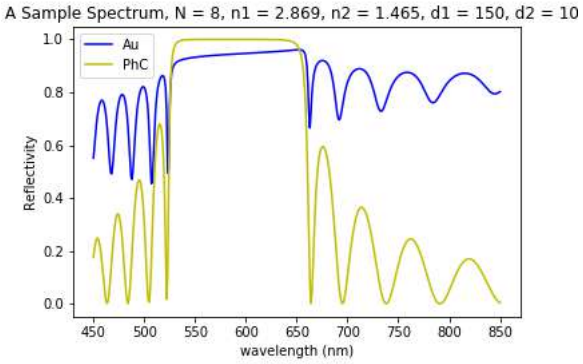


The first normal mode after the metal is coated on left is shown below. The electric intensity pattern is the same as that without metal coating but with a smaller magnitude.



Note that the incidence surface has a lower refractive index of the two material constituting the bilayers. The metal is coated to the surface with the lower refractive index so as to obtain a Tamm state if the light has to be incident on the metal coating.

Now let us consider the case when metal is coated on the surface of higher refractive index and light is incident on it. No Tamm state is formed in this case. The effect of metal coating just increases the reflectivity of PhC as shown in the following figure. Note that $n_1 = 2.869 > n_2 = 1.465$, There is no Tamm state in this case.



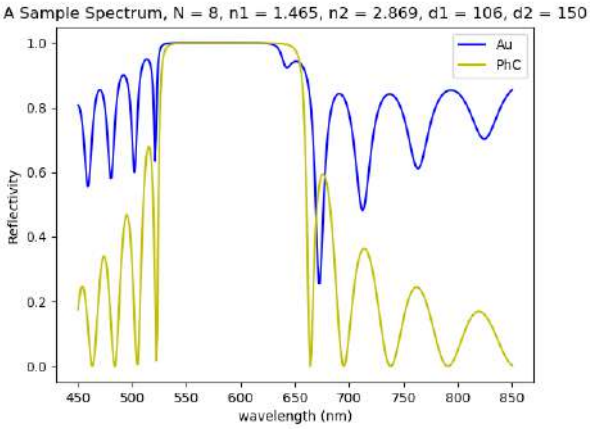
We thus conclude that to a Tamm state the metal has to be coated on the material of lower refractive index if the light is incident on the metal coating.

3.3.2 Metal coated opposite of the incidence surface.

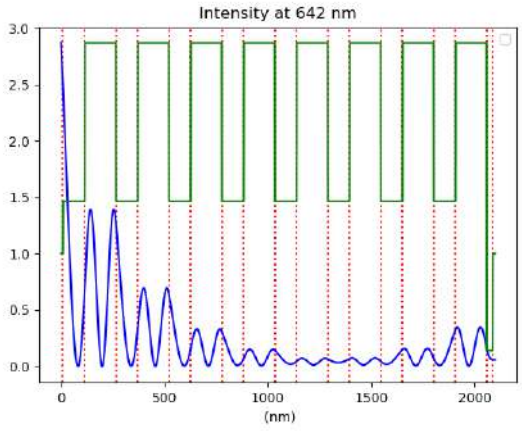
Now, the metal is coated on the surface opposite to the one on which light is incident. The configuration is as follows

$$\text{White light}\rightarrow \left| n_1 \right| n_2 \left| n_1 \right| n_2 \left| \ldots \right| n_1 \left| n_2 \right| \text{metal} \left|\right.$$

If $n_1 < n_2$, that is, the metal is coated on the higher refractive index material, the spectrum obtained for the PhC considered is

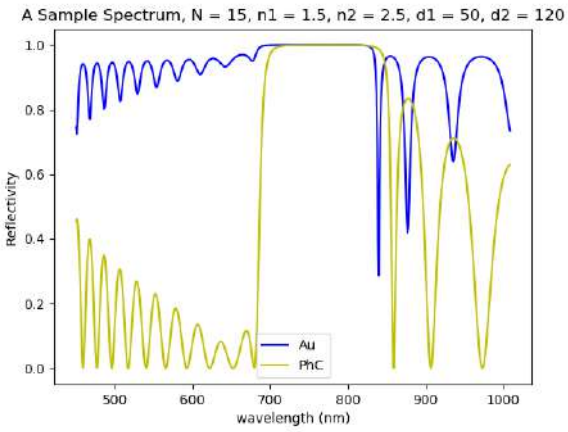


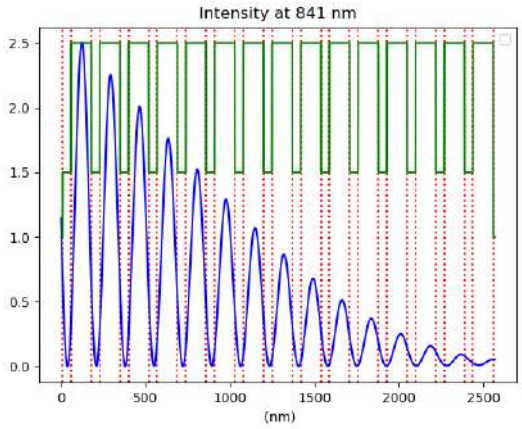
Note the Tamm state formed at the interface of metal and PhC as shown in the following figure

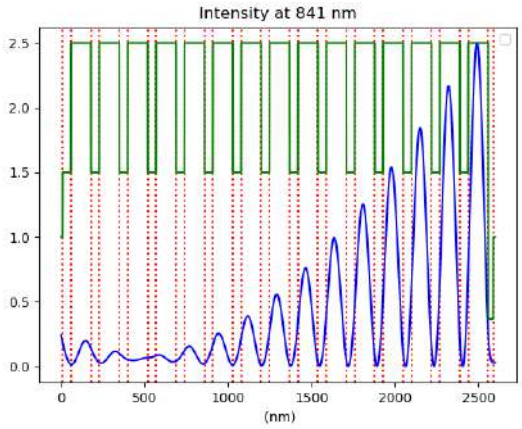


In this figure, we note that observe that the Tamm state formed has a very low dip in the reflectivity spectrum. As it is evident from the plot of field intensity, on the metal-PhC interface number of photons is comparatively higher. To emphasize the fact that a Tamm state is formed at the metal-PhC interface, we choose a different set of parameters and observe the field distribution before and after metal coating.

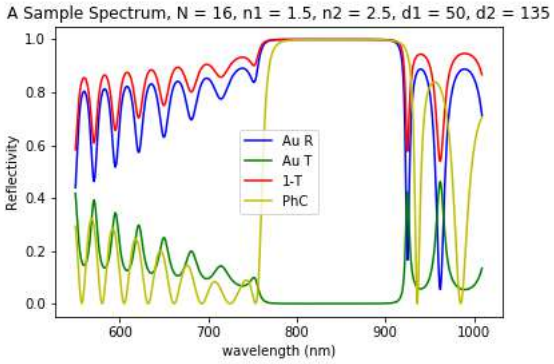
We changed the plasma frequency for the following three plots. So the metal is no more Gold, but some different metal. Following is the spectrum and the fields at the Tamm state wavelength before and after metal coating.







From these simulations, it is evident that the Tamm state are formed at the metal-PhC interface. It is important to observe that the metal is coated on the higher refractive index material when the incidence surface is opposite of metal coating. Let us analyze what happens if the metal is coated on the material with the lower refractive index.



Thus we observe that in order to get a Tamm state when the light is incident opposite to the metal coated surface, the metal has to be coated on the higher refractive index material.

Tamm states were identified from the electric field intensity distribution inside the PhC. We found the condition for observing the Tamm state. If the light is incident on the metal surface then the metal has to be coated with the lower refractive index material. On the other hand, if the metal is on the opposite surface of the incident surface, it has to be coated to the material with a higher refractive index.

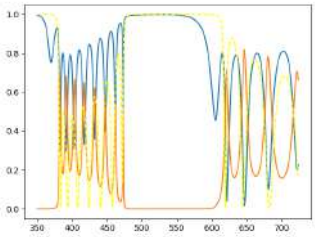
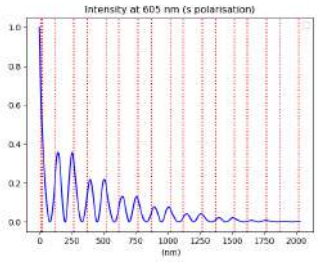
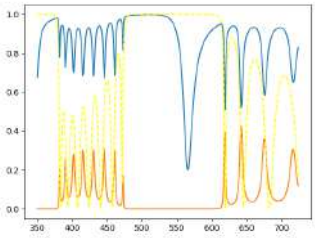
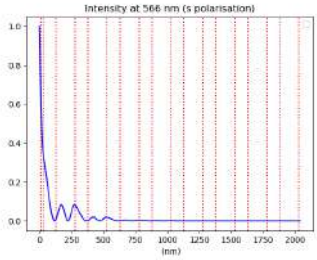
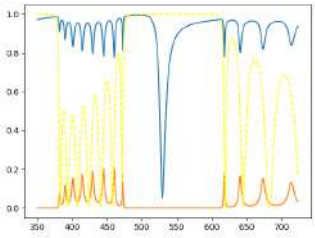
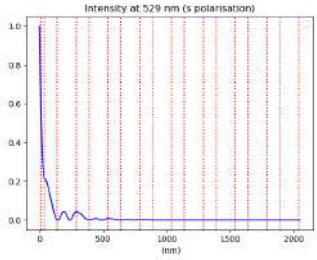
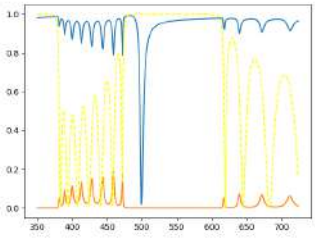
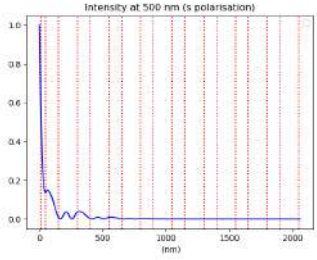
3.4 Effect of varying metal thickness

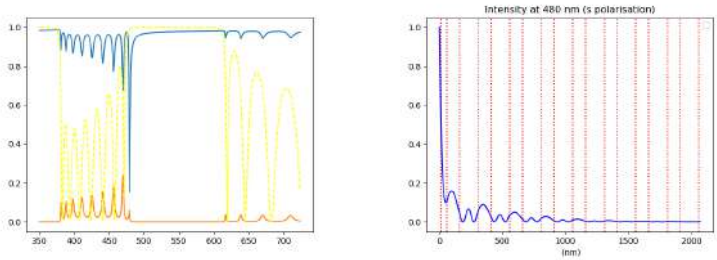
Let the light is incident on the Au-coated surface of the photonic crystal with an angle of incidence of 60^o degree. The PhC is of 8 bilayers with refractive indices $n_1 = 1.500$ and $n_2 = 2.874$ with thicknesses 100 nm and 150 nm respectively. The metal coated photonic crystal has the following arrangement.

$$\text{Light}\rightarrow \left| \text{ Au } \right| n_1 \left| n_2 \right| n_1 \left| n_2 \right| \ldots \left| n_1 \right| n_2 \left|\right.$$

The following plots are for reflectivity and transmissivity spectrum for metal thicknesses 10nm, 20nm, ... 60 nm. The left column represents the spectrum while that in the right column is the field distribution for the contained Tamm state.

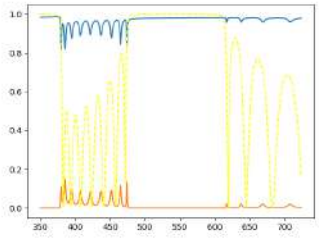
The reflectivity and transmissivity of Au-coated DBR is given by the blue and orange plot. The yellow dotted line represents the reflectivity spectrum of uncoated DBR.



Intensity at 605 nm (s polarisation)
(nm)


Intensity at 566 nm (s polarisation)
(nm)


Intensity at 529 nm (s polarisation)
(nm)


Intensity at 500 nm (s polarisation)
(nm)



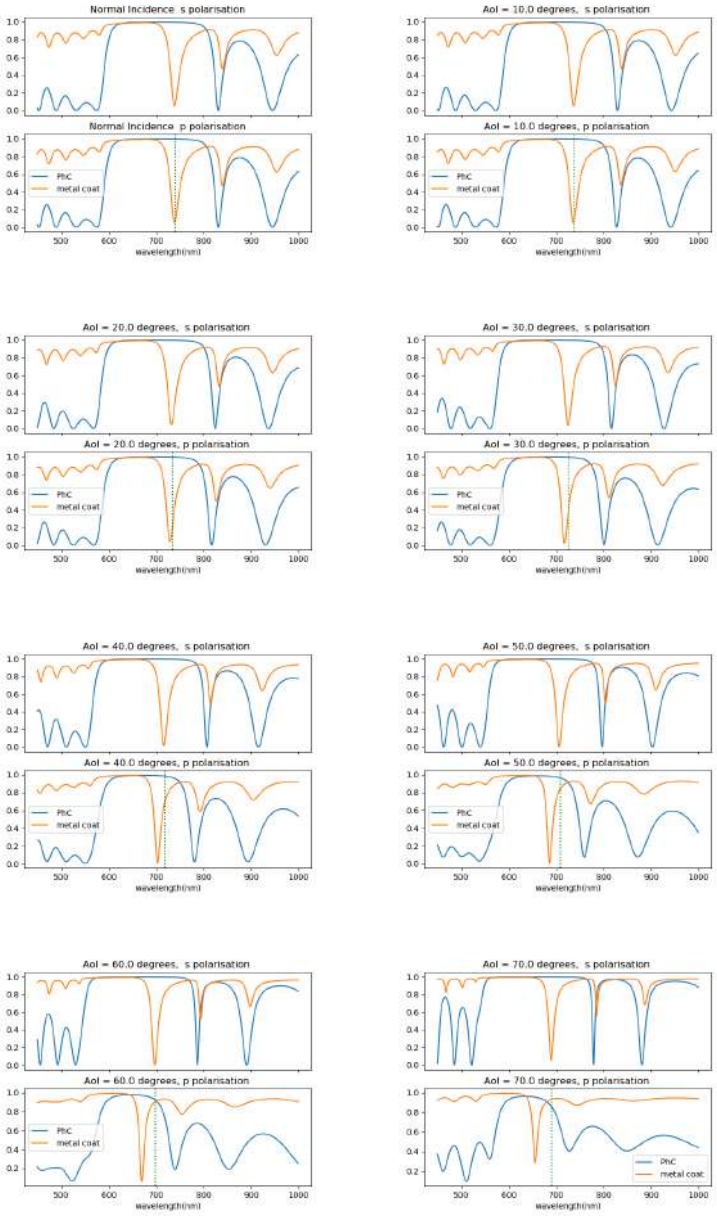
With increase in metal thickness the tamm state wavelength decreases. Equivalently we can conclude that the tamm state energy increases. But the tamm state has to be within the band gap which is determined by the refractive indices and the thickness of DBR layers. So for a given DBR, there exists an upper and a lower bound to the metal thickness for which Tamm state can be observed. The following plot shows the spectrum for 60 nm Au thickness coated on above DBR. We don't observe any Tamm state in this case. Note that the overall reflectivity increases with an increase in the metal thickness, as expected.

These plots were for the 's' polarisation. But it is also true for the case of 'p' polarisation as well.

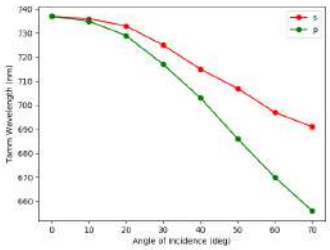


3.5 Tamm resonance wavelength: Variation of the angle of incidence

We consider the above metal-DBR reflectivity spectrum with a fixed metal thickness of 30 nm. We then change the angle of incidence from 0 to 70 degrees in steps of 10 degrees. T



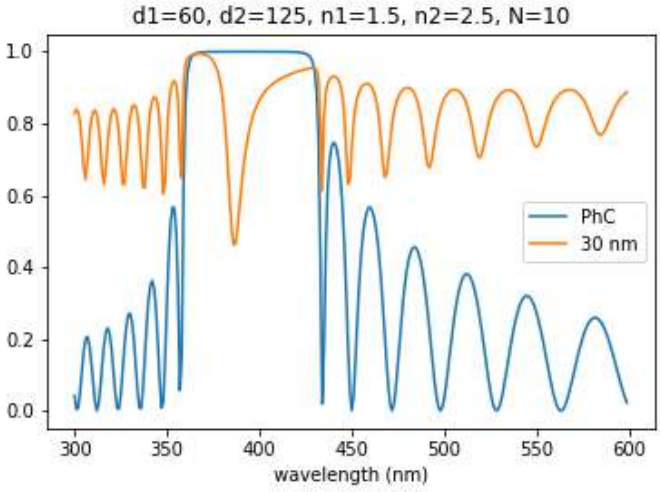
The above plots show Tamm states for both polarisations at the different angle of incidences. The vertical line on the 'p' polarisation plot corresponds to the Tamm state of corresponding 's' polarisation.



The above plot shows how the Tamm wavelength varies with increase angle of incidence. As is clear from the plots the Tamm resonance wavelength decreases with an increase in angle of incidence, and hence the energy increases. Moreover, it is important to note that the wavelength of Tamm state for 'p'-polarisation decreases faster than that of 's'-polarisation wavelength.

3.6 Tamm State for Broken Symmetry in DBR

In the following case, we add a layer to the incident surface consisting of higher refractive index material and the metal, with metal towards left. The total thickness of the layer is that of n_2 medium inside the PhC. The plot of reflectivity, in this case, is following.

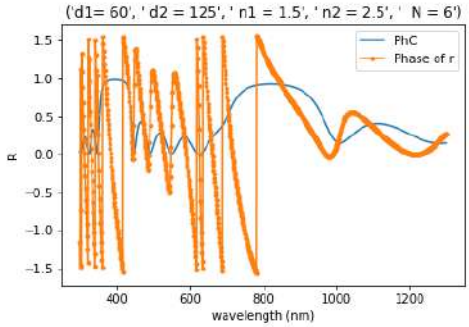


The tamm state in this case is at 386 nm. The parameters, in this case, are shown clearly in the plot. But there is a layer of 125 nm to the left of the leftmost layer of photonic crystal, due to which a Tamm state is formed. There

is a metal width of 30 nm and then n_2 material with width 95 nm. And then the periodic PhC starts. The impact of this broken symmetry is seen in the phase of the coefficient of reflection.

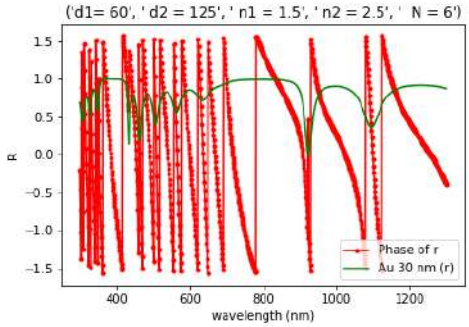
3.7 Phase of Coefficient of Reflection

For normal PhC the phase of coefficient of reflection versus wavelength of light is shown in the following plot.



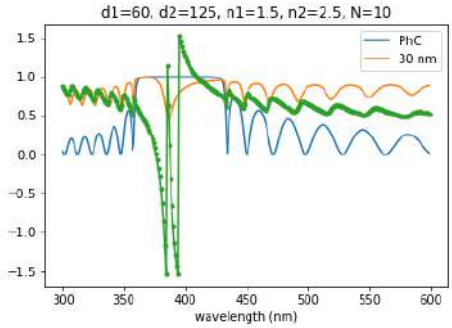
The phase varies from $\pi/2$ to $\pi/2$ in the region of the band gap. In another region of wavelength, the phase difference is finite. In fact, the phase is continuous of the wavelength range is not in the bandwidth.

Not if we coat metal (right of PhC), we observe the following plot



Note at all points of local minima of reflectivity, there is a phase discontinuity of magnitude pi(from $-\pi/2$ to $\pi/2$).

If the symmetry is broken then we have the following phase plot.



in this case phase is continuous everywhere except for the tamm state.

These were the results of computation we studied computationally. We can explore more, and study other quantities such as the phase of transmission coefficient, electric field distribution for 'p' polarised light, the variation of a spectrum with the refractive index of one of the material, variation with the thickness of one media, variation with the number of bilayers etc. But for the experiment we are concerned with now, we require only these results for applications.

We shall use this program to characterize the spectrum of given DBR. In the following section, we shall present our experimental work and show how the computational results and experimental results match with each other.

4 Experiment

We performed the following set of experiments to study and verify the concepts of developed theory in practice, and to verify our computational results. We used our program to characterize a given DBR and found it's parameters. The final experiment was to study the dependence of Tamm resonance wavelength on the angle of incidence for both s and p polarisations. The experimental results were also compared with the ones obtained computationally.

4.1 Experimental Setup

Following apparati were used to setup the experiment

1. White Light Source
2. Collimating Convex Lens

3. Aperture

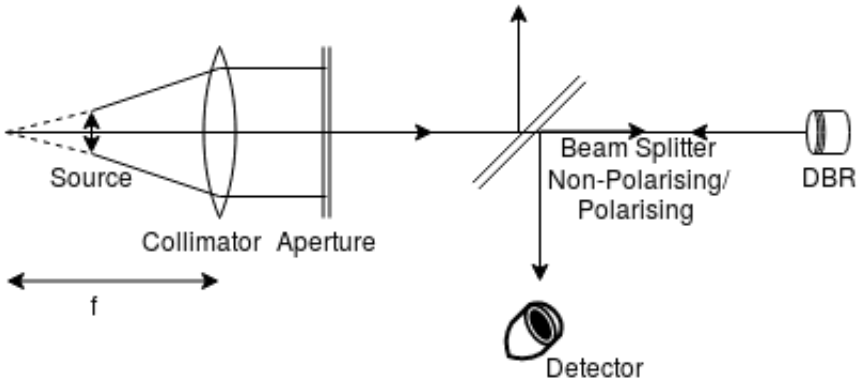
4. Non-Polarising Beam Splitter and Polarising Beam Splitter

5. Uncoated and Coated DBR (with stand of rotatable base)

6. Spectrometer

7. Stands and Breadboard

The schematic ray diagram of our experimental setup is given below

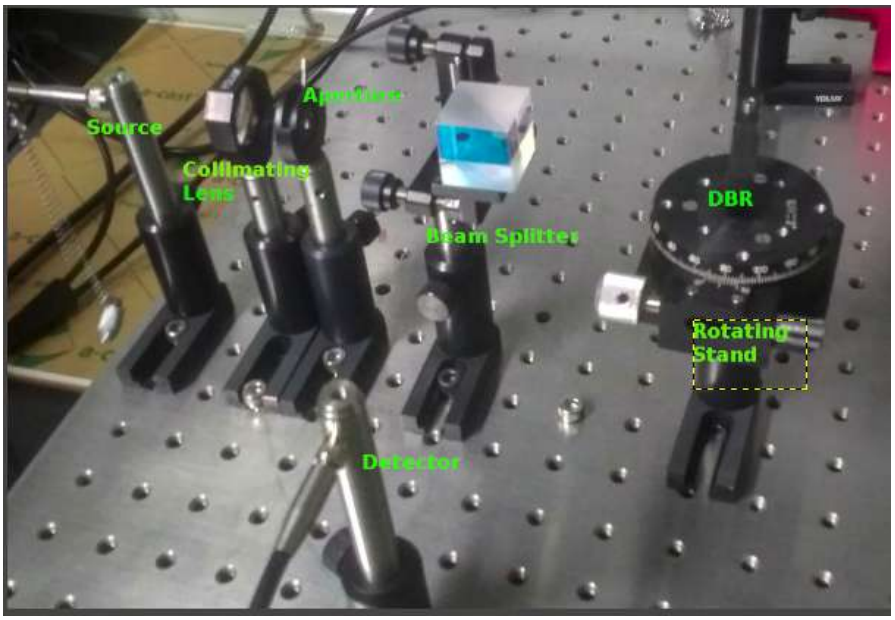


4.2 Methodology

The source of the white light is NOT a point source. It gives a diverging beam. Hence, the virtual point source of this beam is taken to be at a distance of focal length of a collimating convex lens. This was done to obtain a parallel beam of light. A beam (of homogenous intensity) of a smaller radius is extracted from this parallel beam using an aperture. This narrow beam of white light is passed through a beam splitter.

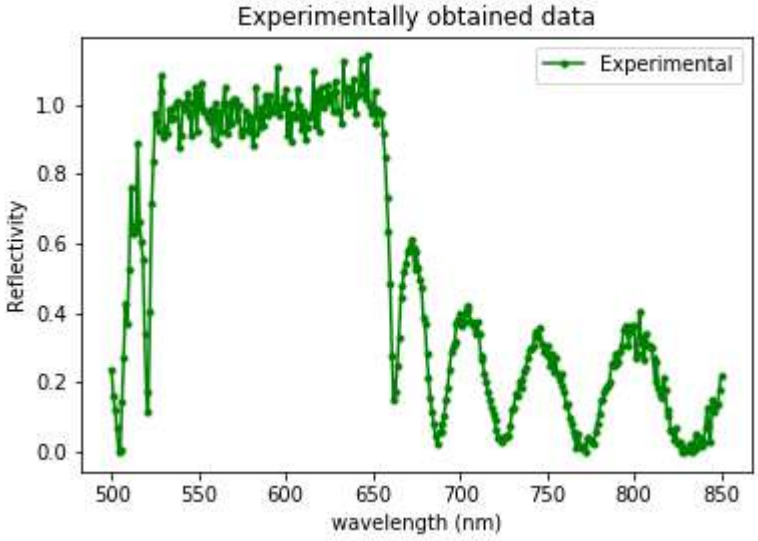
Depending on experimental requirement we use either polarising or non-polarising beam splitter. For Non-polarising beam splitter(NBPS), the transmitted wave is used as incident to the DBR surface and the reflected beam from DBR is further reflected by the same NPBS to the detector (spectrometer). The reflection spectra is analysed in a software BWtek. For transmission spectra, the detector is placed behind the DBR. For a Polarising Beam splitter, the only difference in the procedure is that the transmitted light from PBS is p-polarised

and the reflected beam is s-polarised. Thus, when one wants to study the phenomena for both the polarisations separately in order to compare them, the incident light to DBR is changed accordingly. The actual setup which was used during the experiment is shown below:



4.3 Experimentally obtained Spectrum

The experimental setup consisted of a white light source which was collimated to a parallel beam. A small aperture was used to take a small cross-section from the beam and was passed through a beam splitter. The non-polarisable beam splitter available in Lab was used for this experiment. The reflected light was analyzed by a spectrum analyzer, and the data for reflectivity spectrum was saved on a computer. Following are the plots for reflectivity, that were obtained experimentally.



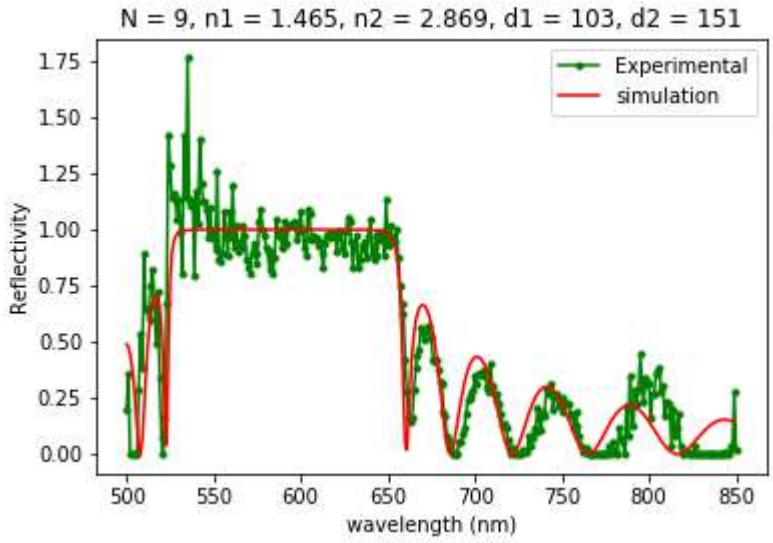
Many of such plots were obtained. We then fitted the plots with the simulation.

4.4 Characterizing a given PhC

AIM : To determine the number of bilayers and the refractive indices and corresponding thickness.

We plotted the above plots simultaneously with the simulated plots and looked for the best-matched simulated spectrum. Then we assigned the parameters used for the simulation to the PhC for which the experimental plot was obtained.

Following are some of the fitting attempts.

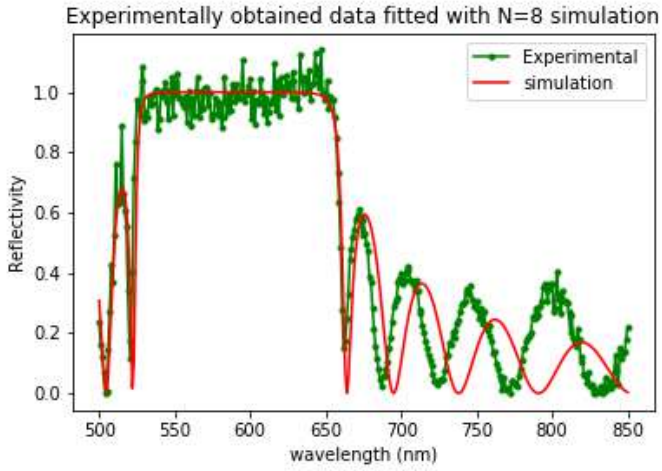


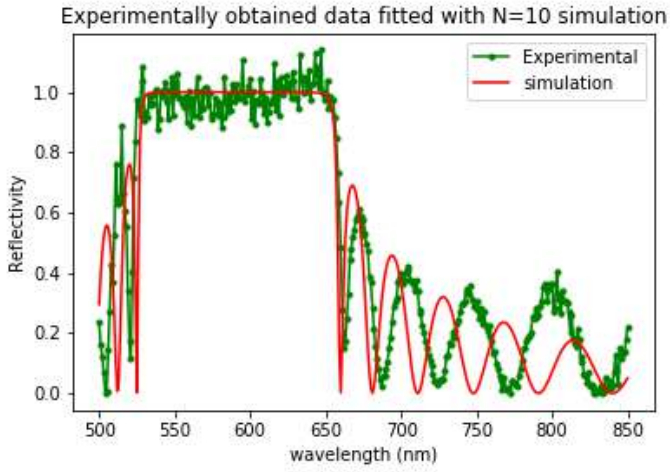
The parameters are mentioned on the top of the plot. According to the above simulated plot, the PhC is of nine bilayers and each bilayers made up of two different materials with refractive indices n_1, n_2 of thickness d_1 and d_2 respectively.

We conclude that the number of bilayers of the given PhC is 9, each consisting of materials of refractive indices 1.465 (SiO 2) and 2.869(T iO 2) of thickness 106 nm and 150 nm respectively. We discussed the fitting scheme of our program. Tamm states were identified from the electric field intensity distribution inside the PhC.

4.4.1 The Fitting method

The number of bilayers is usually less than twenty and is obviously an integer. So N is changed manually. For the purpose of demonstration, we show the plots for $N = 8$ and $N = 10$.





Note that the spectrum for $N = 8$ or $N = 10$ does not match well after the band gap. Hence $N = 9$, as can be seen clearly, is the best fit for the spectrum.

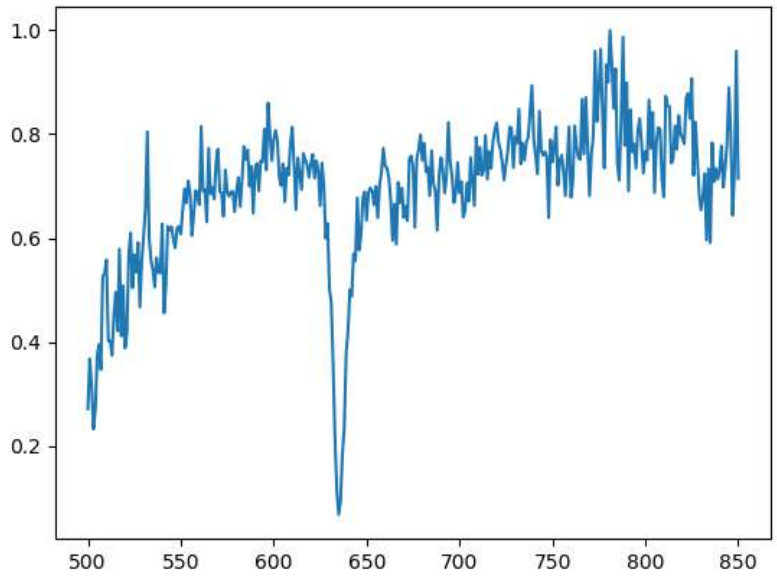
Typically the refractive indices of material which make up the PhC bilayers are known. In our case they are TiO_2 and SiO_2 with refractive indices 2.8 and 1.5 respectively. But a slight variation can be introduced so as to get the fit. Having matched N, it requires some manipulation of d_1 and d_2 to get an appropriate fit. Once the parameters are obtained, the same parameters are used to fit other experimental plots. If it matches for almost all further experimental plots, then we select those parameters as the optimum ones.

To summarise we obtained the following parameter for the PhC

1. No. of bilayers : 09
2. Refractive index, n1 : 1.465
3. Refractive index, n2 : 2.869
4. Width of n1 layer : 106±3 nm
5. Width of n2 layer : 150±3 nm

4.5 Observation Of Tamm Resonance Wavelength

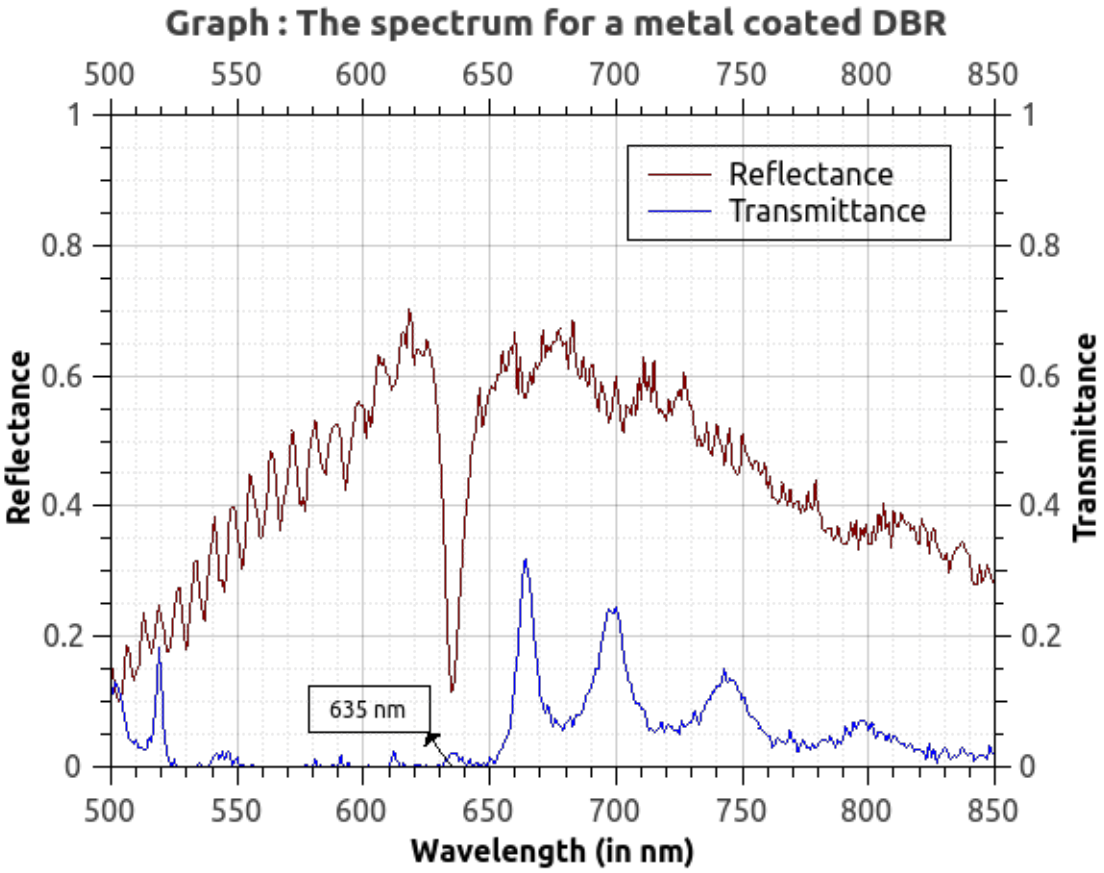
AIM : Qualitative observation of the Tamm state and determination of the resonant wavelength. Here also NPBS was used and the methodology was followed. We got the following observation while taking the spectrum of the first sample.



There is a dip near 635 nm, which can be suspected as a Tamm state. In order to confirm it to be a Tamm state, the transmissivity spectra have to be

obtained. if the transmissivity is lower at this wavelength then we are assured that it is a Tamm state.

For the following graph, it is confirmed that the 635 nm is the Tamm resonance wavelength.



4.6 Tamm resonance wavelength and Angle of Incidence

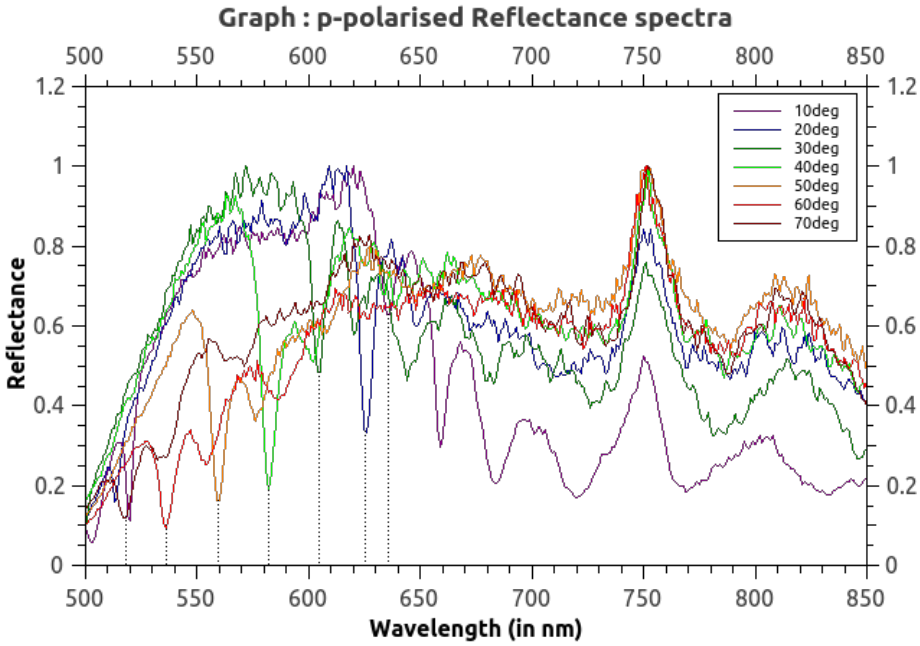
AIM : Dependence of Tamm Resonant Wavelength on Incident Angles for both the polarisations and compare the two.

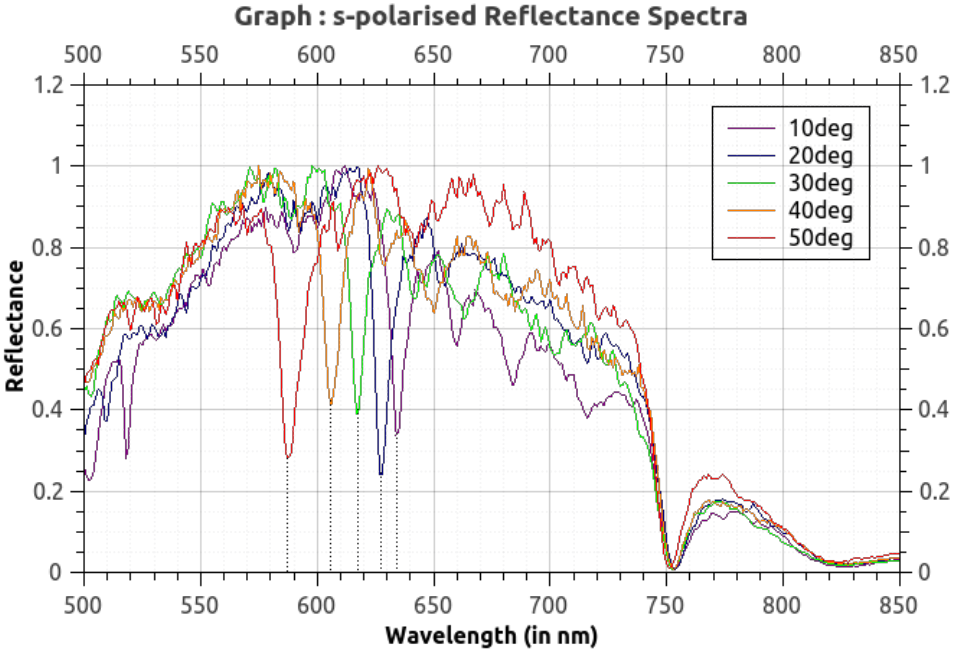
OBSERVATIONS AND RESULTS

The methodology is followed but, the metal-coated DBR (30 nm Au) was placed on a rotating base and the Polarising Beam Splitter is used. The transmitted light from the Beam-splitter is p-polarised while the reflected light is s-polarised. Thus the spectrometer and the DBR are placed accordingly.

The obtained data for the two polarisations are given below. The verification of Tamm signature (High Absorption) is done for every observation recorded (Appendix) and in the graph, the respective resonant wavelengths marked in dotted line.

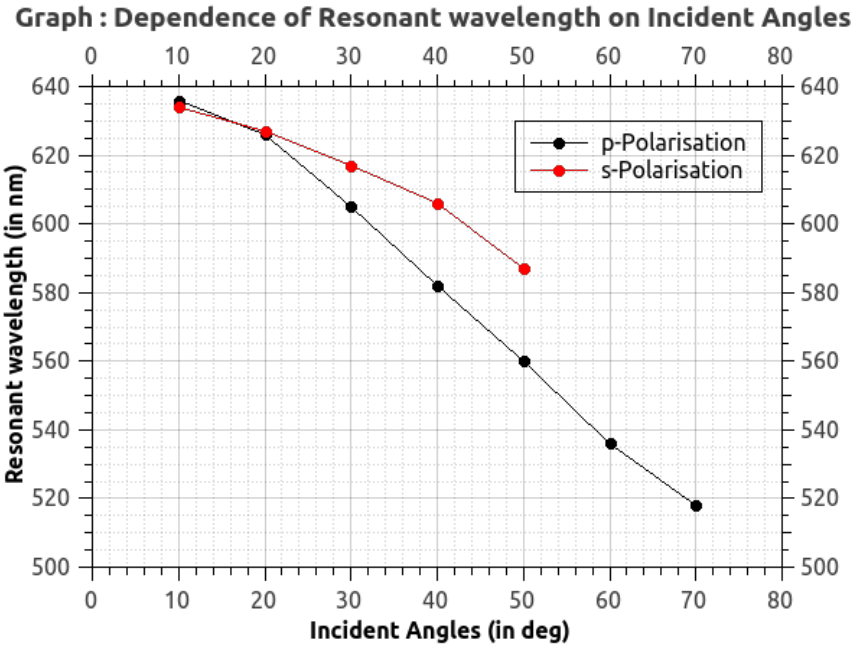
Angle of Incidence (degree)	p-polarised (nm)	s-polarised (nm)
10	636	634
20	626	627
30	605	617
40	582	606
50	560	587
60	536	
70	518	





We see that with an increase in the angle of incidence the Tamm resonance wavelength decreases, as expected from the computational results.

The following experimentally obtained plot shows the variation of Tamm resonance wavelength with the angle of incidence.



It is evident that the Tamm resonance wavelength of p-polarisation decreases faster with increasing angles than the s-polarisations. The lower wavelength will correspond to TM mode and the higher one will correspond to TE mode. For a non-polarised light, we obtain the reflectivity spectrum with two dips. Since the p polarised light decreases faster in Tamm wavelength, the state with the lower wavelength in amongst the two dips corresponds to TM mode Tamm state, while the other one corresponds to the TE mode Tamm state.

5 Future Prospect

Having studied optical surface states both computationally and experimentally, we are in a position to extend this experiment to explore further. We now have a properly working program DBR Spectra, whose results have been experimentally validated as well as confirmed from literature. Using the program to it's fullest potential allowed us to predict some results which were not yet experimentally tested. We briefly discuss some of the future plans which we want to carry out.

5.1 Phase Calculation

The research article "Measurement of reflection phase using thick-gap Fabry–Perot etalon" by Tsz Kit Yung et al. describes how to measure the reflection phase of the interface between a DBR and metal.

The authors report measurement of the reflection phase of a dielectric(glass)/metal(titanium) surface in the visible wavelength using a thick gap Fabry-Perot interferometry technique. They use a two-beam interference model for the reflection peaks and trough for FP etalon[9].

We shall simulate their results and try to replicate this experiment using the metal coated DBR and see if the relevant plots are obtained or not. If so, then we can, at least in principle, calculate the relative phase of reflection at the interface.

5.2 Experimental verification of Computational Results

We can verify some of the computational results. If we are given access to metal deposition laboratory, then the metal thickness on the DBR can be varied and all the plots presented in the report could be realised.

Similarly, if symmetry breaking is feasible, then, other properties like finding the exact impedance matching condition could be verified.

If we are provided with a fresh DBR, then we shall characterize it properly using DBR Spectra, then layer by layer properties concerning metal thickness can be studied. Some accidental observation of Goos-Hanschen shift can also be proved or disproved.

5.3 Sensitive singular-phase optical detection

The article titled, "Sensitive singular-phase optical detection without phase measurements with Tamm plasmons" by Svetlana V. Boriskina and Yoichiro Tsurimaki, the design strategy to achieve perfect absorptance in Tamm structures with dissipative losses via conjugate impedance matching' has been outlined. They, further demonstrate via modeling 'how these structures can be engineered to support sharp asymmetric amplitude resonances, which can be used to improve the sensitivity of optical sensors in the amplitude-only detection scheme that does not require use of bulky and expensive ellipsometry

equipment'. This article is very insightful in Symmetry breaking of the crystal. We shall work to simulate their results. One of the motivating factors for studying the breaking of symmetry in the report is this article. As they claim, this impedance matching condition could be exploited to give sensitive optical detection.

Studying Tamm plasmon states, hence is quite interesting, in the theoretical, computational and experimental aspects. The scope of these experiments is very broad. There are many applications as well as interesting phenomena yet to be explored.

6 Conclusion

We started our exploration with the analogy between band structure in condensed matter physics and photonic band gap in optics. We conclude that this analogy can be exploited to determine condensed matter results from optics experiments. Hence for further studies, we worked on computation and experimentation of the optical phenomena using DBRs.

We made a program, named DBR Spectra and presented it's computational principle and technique used. We have properly documented the code organization and how to use the program. This documentation can be used as a user manual for the program. This document will be evolving with changes and modifications made in the program, which will be reported in biweekly reports.

We used our program to study the reflectivity spectra of Photonic Crystal. We simulated the Tamm state by computationally simulating Gold metal coat on the surface on which light was incident. The electric field intensity distribution of these states gave useful insight as to when the Tamm state is formed. We found the condition for observing the Tamm state. If the light is incident on the metal surface then the metal has to be coated with the lower refractive index material. On the other hand, if the metal is on the opposite surface of the incident surface, it has to be coated to the material with a higher refractive index. We also studied the effect of variation of metal thickness and that in the angle of incidence. We found that with an increase in these parameters the Tamm state energy increases.

On the experimental part, we characterized a given photonic crystal by fitting it's reflectivity spectra with the simulated one. We then excited the Tamm

state and determined the resonance wavelength. We then extended our study to obtain the Tamm state for both s and p polarisations and compared the variation of Tamm state in both cases. The simulated results matched with the one experimentally. Both by simulation and experimentation we conclude that the Tamm resonance wavelength decreases with increase in the angle of incidence. The decrease is faster in the case of p polarisation as compared to that in s polarisation.

We briefly outlined the future plans about the computational and experimental approach to the study of optical surface states.

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