

RAJA RAMANNA CENTRE FOR ADVANCED TECHNOLOGY
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Electron-Positron Pair Creation in Strong Laser Fields

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Given the vacuum state, what is the probability that the final state is that of an electron?

Conservation laws will ensure creation of a positron.

We need to understand the vacuum state first. . .

Understanding Vacuum

- ▶ Here's the story

Dirac was trying doing something interesting. . .

Understanding Vacuum

- ▶ Dirac was trying to unify Special Theory of relativity with Quantum Mechanics.
- ▶ According to special theory of relativity(in natural units)

$$E^2 = p^2 + m^2$$

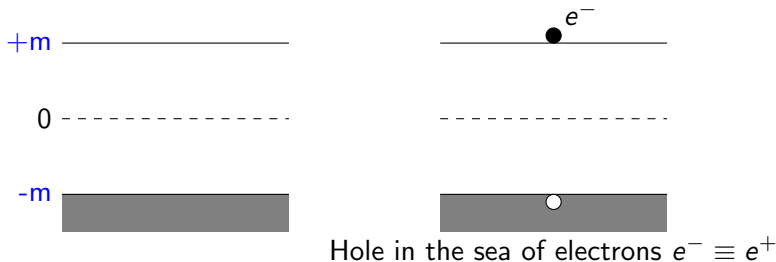
$$E = \pm \sqrt{p^2 + m^2}$$

where E , $p(=|\vec{p}|)$ and m are the Energy , momentum and mass of the particle.

Problem

There are unbounded negative energy states.

Dirac's Picture of Vacuum



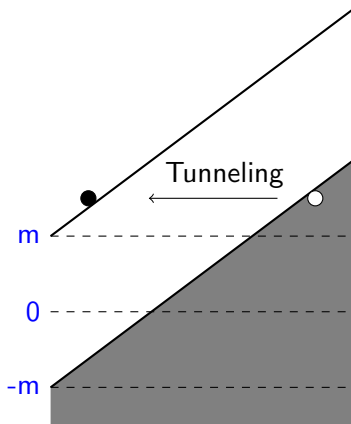
- ▶ The negative energy states are already filled with electrons.
- ▶ The minimum energy required for a negative energy state electron to go into positive energy state is twice the mass of an electron.

How is this energy given?

Regimes of Pair Production in Laser field

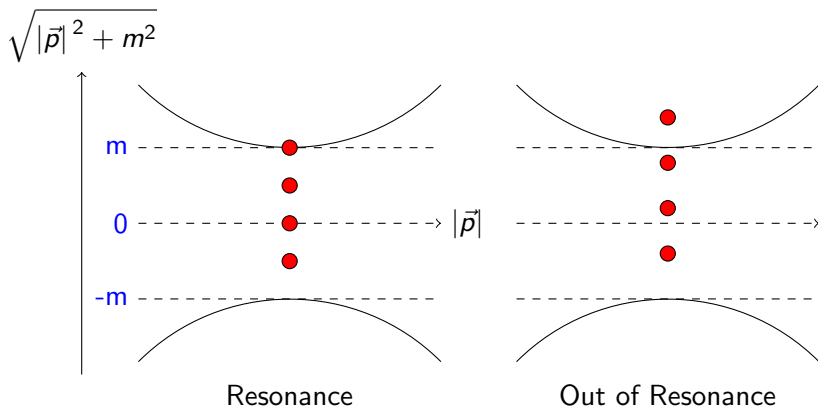
It is an interplay of Laser intensity and Frequency

- High electric field - Tunneling Regime



Regimes of Pair Production in Laser field

- ▶ Low electric field - Multiphoton Regime
- ▶ High Frequency field, i. e. few photons field



Quasiclassical Estimate

- ▶ The wavefunction in quasiclassical approximation is given by

$$\psi_{\pm} \propto \exp\left(\frac{iS_{\pm}}{\hbar}\right) \quad (1)$$

Here, $S_{\pm}$ is the action of positive/negative energy states of electron.

- ▶ The fields we shall take is purely time dependent. This makes $S_{\pm} = \vec{p} \cdot \vec{r} \mp S_0(\omega t)$ where $S_0(\eta) = (1/\omega) \int_{\eta_0}^{\eta} d\eta' [(\vec{p} - e\vec{A})^2 + m^2]$
- ▶ The transition amplitude is given by

$$\begin{aligned} \psi_{-}^{\dagger} \psi_{+} &\propto e^{i(S_{+} - S_{-})} \\ &= \sum_n C_n e^{in\omega t} e^{-2iq_0 t} \end{aligned}$$

where q_0 is the quasistatic energy of electron and C_n are the fourier coefficients of periodic part of transition amplitude.

Finding Pair Production Probabilities in Quasiclassical Approximation

- ▶ The fourier coefficients $\mathcal{C}_n$ determine the probabilities of n-photon process

$$W_n \propto |\mathcal{C}_n|^2$$

- ▶ Given a purely time dependent electric field, one should be able to get the fourier coefficients $\mathcal{C}_n$.

OR

- ▶ $\mathcal{A}_n = e^{i(\text{Constant phase})} \mathcal{C}_n$.

For Example

Consider Rotating Electric Field

- ▶ $\vec{E} = \partial \vec{A} / \partial t$, where the vector potential is

$$\vec{A}(\eta) = \frac{m\xi}{|e|} (\hat{x} \cos \eta + \hat{y} \sin \eta) \quad (2)$$

where $\eta = \omega t$, ξ is the field strength parameter such that $m\xi\omega/|e|$ is the peak electric field.

For this Field there exist an expression for the fourier coefficients $\mathcal{A}_n$

The fourier coefficients are given by...

$$\mathcal{A}_n^{(c)} = \frac{1}{\pi} \int_0^\pi d\eta \exp \left(2iQ \left[E(\eta|\mu) - 2E(\mu) \frac{\eta}{\pi} \right] + 2im\eta \right)$$

where

$$Q = \frac{2}{\omega} \sqrt{(p_\perp + m\xi)^2 + p_z^2 + m^2}$$

$$\mu = \frac{4m\xi p_\perp}{[(p_\perp + m\xi)^2 + p_z^2 + m^2]}$$

and

$$E(\eta|\mu) = \int_0^\eta d\phi' \sqrt{1 - \mu \sin^2 \phi'}$$

with $E(\mu) \equiv E(\pi/2|\mu)$

Results of Asymptotic Analysis

By using Saddle Point Approximation

- ▶ In high electric field strength ($\xi \gg 1$), the probability of pair production is

$$W \propto \exp(-\pi E_c/E)$$

- ▶ In low electric field $\xi \ll 1$,

$$W \propto \xi^{2n_0}$$

where $n_0 = 2q_0/\omega$

Quantum Mechanical Analysis

We shall discuss two techniques

1. Bogolyubov Transformation
2. Free Basis Expansion

Bogolyubov Transformation Technique

Consider the Hamiltonian of the problem ...

$$H(t) = \int d^3x \Psi^\dagger(x) \mathcal{H}(t) \Psi(x) \quad (3)$$

where

$$\mathcal{H}(t) = \vec{\alpha}[-i\vec{\nabla} - e\vec{A}(t)] + \beta m$$

Since canonical momentum is conserved, we analyse the problem in a subspace of a definite momentum $\vec{p}$. The field operator is written as

$$\Psi(x) = \frac{e^{i\vec{p}\cdot\vec{r}}}{\sqrt{V}} \Phi(t)$$

With this

$$H(t) = \Phi^\dagger(t) \mathcal{H}_{\vec{p}}(t) \Phi(t)$$

where $\mathcal{H}_{\vec{p}}(t) = \vec{\alpha}[\vec{p} - e\vec{A}(t)] + \beta m$

Bogolyubov Transformation Technique

The operator $\tilde{\Sigma} \equiv -i\beta\alpha_x\alpha_y$

$$\tilde{\Sigma} = \begin{bmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{bmatrix}$$

- ▶ It is a 4-Dimensional Matrix.
- ▶ It commutes with $\mathcal{H}_{\vec{p}}(t)$.
- ▶ It has eigen values $+1$ and -1 , with eigen spaces $|+1\rangle$ and $|-1\rangle$ given by

$$|+1\rangle = \begin{bmatrix} \Phi_1(t) \\ 0 \\ 0 \\ \Phi_2(t) \end{bmatrix} \text{ and } |-1\rangle = \begin{bmatrix} 0 \\ \Phi_1(t) \\ \Phi_2(t) \\ 0 \end{bmatrix}$$

We shall work in the subspace $|+1\rangle$

Finding Probability of Pair Production

Suppose the field is switched on at $t = t_{in}$ and switched off at $t = t_{out}$. Define IN and OUT operators as follows

$$\Phi(t) = a_{in}\Phi_{in}^{+}(t) + b_{in}^{\dagger}\Phi_{in}^{-} = a_{out}\Phi_{out}^{+}(t) + b_{out}^{\dagger}\Phi_{out}^{-}$$

where $\Phi_{in}^{\pm}(t)$ are exact solutions of Dirac Equation, which obey the boundary condition of being positive and negative energy eigen spinors of $\mathcal{H}_{\vec{p}}(t)$ at $t < t_{in}$.

- ▶ a_{in} annihilates an electron. $a_{in}^{\dagger}$ creates electron.
- ▶ b_{in} annihilates a positron. $b_{in}^{\dagger}$ creates a positron.

Finding Probability of Pair Production

The IN and OUT creation and annihilation operators are related by Bogolyubov transformation

$$\begin{aligned}a_{out} &= c_2 a_{in} - c_1^* b_{in}^\dagger \\ b_{out}^\dagger &= c_1 a_{in} + c_2^* b_{in}^\dagger\end{aligned}$$

with $|c_1|^2 + |c_2|^2 = 1$. The pair production probability is thus given by

$$w = \langle 0_{in} | a_{out}^\dagger a_{out} | 0_{in} \rangle = |c_1|^2 \langle 0_{in} | b_{in} b_{in}^\dagger | 0_{in} \rangle = |c_1|^2$$

The Hamiltonian can be diagonalised at every moment during interaction with proper Bogolyubov Transformation

$$\begin{aligned}a(t) &= g^*(t)a_{in} + f(t)b_{in}^\dagger \\ b^\dagger(t) &= -f^*(t)a_{in} + g(t)b_{in}^\dagger\end{aligned}$$

such that $|f(t)|^2 + |g(t)|^2 = 1$

The probability of pair production is given by

$$W = |f(t)|^2$$

For a given pulse of duration $t_{out} - t_{in}$, $|c_1|^2 = |f(t_{out})|^2$

Time Evolution of $f(t)$ and $g(t)$.

One can find differential equations for f and g which are given by

$$f'(t) = ic(t)f(t) + d(t)g(t)$$

$$g'(t) = -d^*(t)f(t) - ic(t)g(t)$$

where $c(t)$ and $d(t)$ are functions of mass and momentum of electron and the applied vector potential.

- ▶ This can be solved numerically. But some analytic work can also be done.
- ▶ We solve this system using the initial condition $f(t_{in}) = 0$.

Results From Analytic Work

Rabi Oscillations at Resonance

- ▶ For an interaction of time $\tau \equiv t_{out} - t_{in}$, the pair production probability is given by

$$W(\tau) = \frac{|b_{n_0}|^2}{\Omega_{n_0}^2} \sin^2 \Omega_{n_0} \tau$$

where $\Omega_{n_0} = \sqrt{|b_{n_0}|^2 + (2q_0 - n_0\omega)^2/4}$ is the rabi frequency.

- ▶ $b_{n_0} \sim \mathcal{A}_{n_0}$. Thus, $W \propto |\mathcal{A}_{n_0}|^2$, as shown in quasiclassical approximation.

The technique of ...

Free Basis Expansion

Free Basis. . .

Consider the Dirac Hamiltonian

$$i\frac{\partial}{\partial t}\psi(x) = \mathcal{H}(t)\psi(x) \quad (4)$$

This equation when solved for $\mathcal{H}(t) = \vec{\alpha} \cdot [-i\vec{\nabla} - e\vec{A}(t)] + \beta m$, with $\vec{A} = 0$, gives the free solutions. Here $x = (t, \vec{x})$ and $p = (E, \vec{p})$.

- Denote the free solutions as

$$\phi_p^r(x), \text{ where } r \in \{1, 2, 3, 4\}$$

Expand wave function (in presence of field) in terms of free basis.

$$\psi_p(x) = f(t)\phi_p^1(x) + c(t)\phi_p^2(x) + d(t)\phi_{p^*}^3(x) + g(t)\phi_{p^*}^4(x)$$

where

$$\phi_p^1(x) = \sqrt{\frac{E+m}{2EV}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ \frac{p_x + ip_y}{E+m} \end{bmatrix} e^{-ip \cdot x} \quad \phi_p^2(x) = \sqrt{\frac{E+m}{2EV}} \begin{bmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ 0 \end{bmatrix} e^{-ip \cdot x}$$

$$\phi_{p^*}^3(x) = \sqrt{\frac{E+m}{2EV}} \begin{bmatrix} 0 \\ \frac{-p_x - ip_y}{E+m} \\ 1 \\ 0 \end{bmatrix} e^{ip^* \cdot x} \quad \phi_{p^*}^4(x) = \sqrt{\frac{E+m}{2EV}} \begin{bmatrix} \frac{-p_x + ip_y}{E+m} \\ 0 \\ 0 \\ 1 \end{bmatrix} e^{ip^* \cdot x}$$

On putting the wavefunction in Dirac equation with field switched on...

We obtain following coupled Differential Equation

$$\begin{aligned}\frac{df}{dt}(t) &= \kappa(t)f(t) + \nu(t)g(t) \\ \frac{dg}{dt}(t) &= -\nu^*(t)f(t) + \kappa^*(t)g(t)\end{aligned}$$

where κ and ν are functions of mass and momentum of electron and the vector potential applied.

- ▶ The Pair Production probability is given by

$$W = |f(t)|^2$$

- ▶ This system of equations can be transformed to a simple form...

A simple looking form

$$\begin{aligned}\frac{d\bar{f}}{dt}(t) &= \zeta(t)\bar{g}(t) \\ \frac{d\bar{g}}{dt}(t) &= -\zeta^*(t)\bar{f}(t)\end{aligned}$$

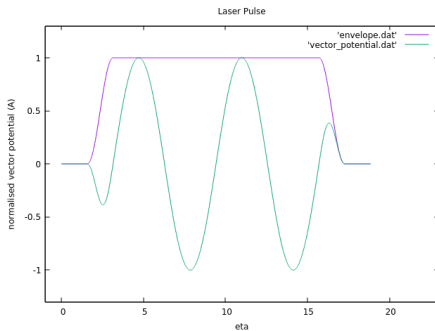
where $\bar{f} = e^{i(\text{phase})}f$, $\bar{g} = e^{i(\text{phase})}g$ and $\zeta(t)$ is a function of electron mass, momentum and of the vector potential.

- ▶ This system of equation was numerically solved for $\vec{p} = 0$, and $\xi = 1$ of an oscillating electric field.

Numerical Studies

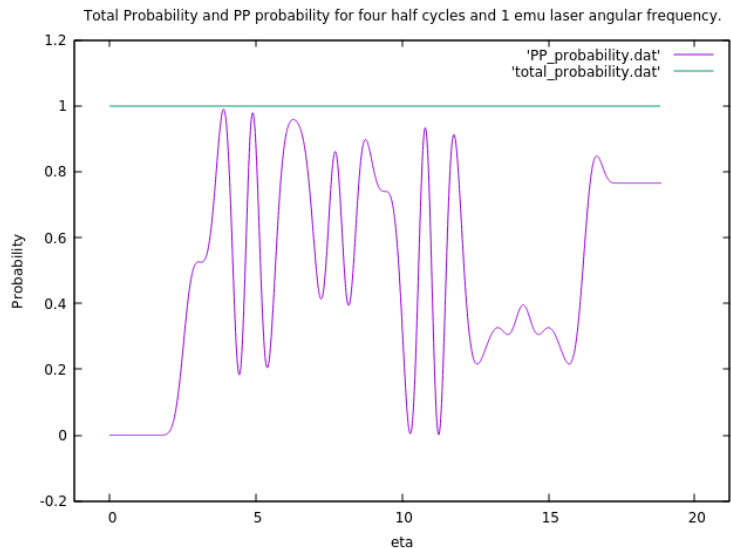
The Vector Potential

The following pulse is applied

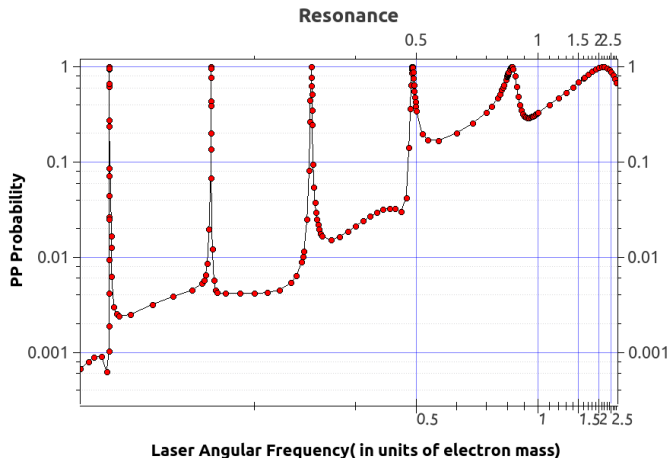


The x-axis represents $\eta = \omega t$.

Evolution of probabilities with time



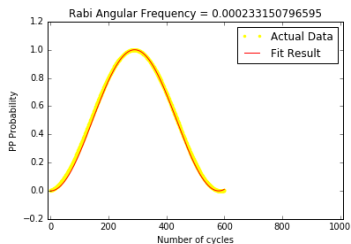
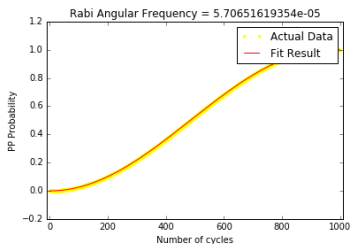
Resonant Frequencies



The resonant frequencies are
0.2214656, 0.270911, 0.38492, 0.49150 and 0.82 in units of electron
mass.

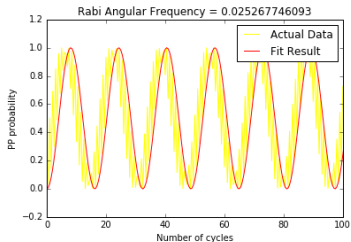
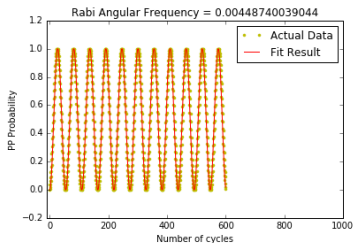
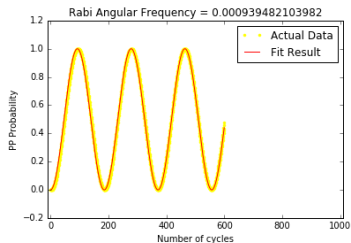
Rabi Oscillations at Resonant Frequencies

- ▶ Rabi Oscillations are observed at resonance frequencies as predicted analytically by using the technique of Bogolyubov Transformation.



- ▶ Rabi Frequency was found by fitting with $\sin^2[(2\pi\Omega/\omega)N]$

Rabi Oscillations at Resonant Frequencies



Summary

- ▶ Using Quasi Classical Approximation we showed that Pair Production depends on the fourier coefficients of the periodic part of transition amplitude, and it was confirmed by Bogolyubov Transformation Technique.
- ▶ Bogolyubov transformation technique analytically predicted Rabi Oscillations at Resonant frequencies of Pair Production.
- ▶ This was numerically confirmed by the technique Free Basis Expansion.

Future Work

We shall further extend the study for a chirped laser pulse. The electric field of the chirp is given by

$$E(t) = E_0 \cosh^{-2}(t/\tau) \cos(\omega_0 t + \beta t^2 + \alpha t^3)$$

where $E_0, \omega_0, \tau, \beta, \alpha$ are constants.

Nonperturbative multiphoton electron-positron-pair creation in laser fields.

R. Mocken, Guido & Ruf, Matthias & Müller, Carsten & H. Keitel, Christoph. (2010). Physical Review A, v.81 (2010). 81.
10.1103/PHYSREVA.81.022122.

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Thank You